

## Using the inverse scattering series to predict the wavefield at depth and the transmitted wavefield without an assumption about the phase of the measured reflection data or back propagation in the overburden

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### ABSTRACT

The starting point for the derivation of a new set of approaches for predicting both the wavefield at depth in an unknown medium and transmission data from measured reflection data is the inverse scattering series. We present a selection of these maps that differ in order (i.e., linear or nonlinear), capability, and data requirements. They have their roots in the consideration of a data format known as the T-matrix and have direct applicability to the data construction techniques motivating this special issue. Of particular note, one of these, a construction of the wavefield at any depth (including the transmitted wavefield), order-by-order in the measured reflected wavefield, has an unusual set of capabilities (e.g., it does not involve an assumption regarding the minimum-phase nature of the data and is accomplished with processing in the simple reference medium only) and requirements (e.g., a suite of frequencies from surface data are required to compute a single frequency of the wavefield at depth when the subsurface is unknown). An alternative reflection-to-transmission data mapping (which does not require a knowledge of the wavelet, and in which the component of the unknown medium that is linear in the reflection data is used as a proxy for the component of the unknown medium that is linear in the transmission data) is also derivable from the inverse scattering series framework.

### INTRODUCTION

There are many fields of nondestructive investigation, of which both exploration seismology and deep-earth investigation are prime

examples, whose aim in broad terms is to make inferences about the interior of an object from external measurements. However, these two examples represent very different types, extent, and completeness of surface measurements. These surface measurement requirements for determining target properties depend upon many factors, including (1) the assumed ability, or inability, to estimate propagation properties in the medium that is above and/or surrounding a target; (2) whether or not that target overburden can be well approximated (e.g., for the purpose of estimating the wavefield at depth) by a medium that supports only one-way or two-way propagation; (3) the level of ambition in identification of a target (e.g., whether the goal is a structure map of interfaces in their correct spatial location, excluding or including evanescent components, or the more ambitious estimation of earth-property changes across those interfaces); and (4) whether or not the algorithm used for target location and identification requires information about medium properties between the target at depth and surface measurements.

The extent and completeness of recorded surface data is often determined by economic and/or natural acquisition constraints. An example of a natural constraint in whole-earth investigation is that the sources are, typically, only located at great depth (e.g., earthquake sources), whereas the receivers are on the surface. In exploration seismology, sources are man made and, along with all receivers, are located on or near the earth's surface. In the exploration case, natural constraints disallow sources or receivers to be placed at great depth.

There are several types of data construction activities pursued in the seismology of exploration and production that can assist in overcoming some of these constraints. Among them are (1) extrapolating and interpolating surface reflection data, (2) mapping from reflection experiment data to transmission experiment data and vice versa, and (3) mapping surface data to data at depth, the latter of which is an essential ingredient in wave-theoretic migration and migration-inversion algorithms. Taking a surface experiment and inferring an

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other type of surface experiment (e.g., reflection-to-transmission or transmission-to-reflection) has several specific motivations, including (1) allowing the direct use of mature subsurface determination methods that are not available for the measured data but that are available for the predicted data type; (2) increasing surface coverage for methods that require and/or benefit from both reflection and transmission data; (3) increasing the illumination of targets; and (4) allowing evanescent wave prediction in a stable and reliable manner.

The diminishment, in recent years, of the apparent separation between whole-earth and exploration seismology, given the differing experimental configurations and objectives of transmission- and reflection-type experiments (Snieder and Levin, 2004), has been influenced by data construction advancements with the introduction of the daylight imaging method for a 1D medium by Claerbout (1968) and its later generalization to 3D acoustic or elastic media by Wapenaar (2003) and Wapenaar et al. (2004). The method, based on reciprocity principles (see Fokkema and Van den Berg, 1993; de Hoop and de Hoop, 2000), describes a way to relate reflection and transmission data and, under certain conditions, to construct one from another. In essence, the method consists of crosscorrelating passive traces from two surface receivers to create the reflection seismogram that would be computed at one of the locations if there were a source at the other. Claerbout's idea has since been rediscovered, extended, and applied to numerous areas of interest. Among them are helioseismology (Duval et al., 1993; Rickett and Claerbout, 1999), ocean acoustics (Roux et al., 2005), ultrasonics (Larose et al., 2004; Malcolm et al., 2004; Weaver and Lobkis, 2004), and seismology (Snieder et al., 2002; Shapiro and Campillo, 2004; Snieder, 2004; Wapenaar, 2004; Sabra et al., 2005; Shapiro et al., 2005).

Building reflection data from transmission recordings and vice versa provides two kinds of benefits. First, as mentioned, a transmission experiment can be translated into a reflection experiment (and vice versa) and applied to the problem at hand with a set of methods which is normally specific to the other geometry. Second, having data from measurements and data from construction allows the development/application of more complex methods for imaging and inversion. The relation between transmission and reflection seismograms is useful, for example, for seismic imaging of the earth's interior using passive recordings of noise sources located in the subsurface. Among the most notable extensions of the daylight imaging principle, we mention the method of interferometric imaging (Schuster, 2001; Schuster et al., 2004), which broadens the previous concept to any number or distribution of sources and to arbitrary reflectivity distributions. Moreover, the method offers the means to migrate free-surface and internal multiples in the data together with the primaries (e.g., Sheng, 2001; Schuster et al., 2004) and to migrate transmitted waves and locate unknown sources in the subsurface from ambient noise or earthquake recordings. Alternative descriptions include crosscorrelation migration (Schuster, 1999; Schuster and Rickett, 2000) or an extended form of autocorrelation migration (Schuster et al., 1997).

There are, furthermore, specific examples in which the ability to relate reflection and transmission data allows methods developed for one type of geometry to be applied to another problem. In a recent project sponsored by the National Science Foundation (NSF), a group from the University of Houston led by A. B. Weglein, together with a group from Indiana University led by G. Pavlis, has undertaken the task of translating some of the seismic-exploration inverse-scattering series methods and algorithms to the deep-earth seismology problem. The first product of this collaboration was a method for

separating the forward- and back-scattered wavefields from earthquake recordings (Fan et al., 2006), which produced encouraging results on field data (Fan et al., 2005). A further notable method inspired by the daylight imaging principles with application to seismic exploration is the virtual source method (Bakulin and Calvert, 2004).

There is another and more recently understood benefit to predicting transmission data from reflection data. The inverse scattering series currently provides a comprehensive, multidimensional direct-inversion method that allows inverse objectives, e.g., free-surface and internal multiple removal, and depth structure maps and nonlinear direct AVO to proceed, all without knowing or ever determining the actual properties that govern wave propagation in the subsurface. Those inverse tasks are achieved sequentially with distinct algorithms corresponding to task-specific subseries. In addition to being independent of any actual subsurface medium properties governing wave propagation, the free-surface and internal multiple attenuation algorithms are independent of whether the earth is acoustic, elastic, heterogeneous, anisotropic, or inelastic (e.g., Weglein et al., 2003); in other words, they are independent of model type. Recently, a task-specific subseries aimed at performing imaging, i.e., locating reflectors in space, has been identified and tested on analytic data for simple 1D-earth examples with encouraging results (Weglein et al., 2000; Shaw, 2001; Shaw et al., 2001; Weglein et al., 2001; Shaw et al., 2002a, b; Innanen and Weglein, 2003; Shaw, 2003; Shaw et al., 2003a, b; Weglein et al., 2003; Shaw et al., 2004; Shaw, 2005; Liu et al. 2005a, b). Liu et al. (2005a, b) have shown the first multidimensional acoustic examples of the methods for determining the correct spatial location of reflectors without knowing or determining the overburden velocity model. As the analysis of Weglein et al. (2000) and Shaw et al. (2004), with respect to the possibility and requirements for a model-type independent imaging algorithm, and the diagrams of describing inverse scattering interactions within the imaging subseries, show, a model-type independent algorithm would require both reflection and transmission data.

Reflection-to-transmission transformations (of importance, then, to the fundamental capability and concepts of velocity independent depth imaging) are, when task-separated imaging methods are not being considered, themselves provided by distinct methods derived from the inverse scattering series. By way of introducing this new framework for data construction, we next highlight some relevant aspects of the daylight imaging data construction method.

The relation between the two data types is derived from reciprocity theorems of the correlation type written for one-way wavefields (e.g., Wapenaar and Grimbergen 1996; Wapenaar, 2004)

$$\int_{z_M=\text{const.}} d^2\mathbf{x}\{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\} \\ = \int_{z_m=\text{const.}} d^2\mathbf{x}\{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\}, \quad (1)$$

where  $P^+$  and  $P^-$  are flux-normalized down going and up going wavefields, respectively. The derivation of equation 1 assumes that the medium is lossless and that evanescent components can be neglected.

The two states  $A$  and  $B$  can be chosen in different ways to derive relations between reflection and transmission responses [for a comprehensive description see Wapenaar et al. (2004)]. Choosing both states  $A$  and  $B$  to represent experiments with the source located at depth  $z_m - \epsilon$  in a homogeneous half-space (see Figure 1) leads to a relationship which allows the construction of the transmission response from reflection data. Let the space coordinates of the sources be denoted by  $\mathbf{x}_A$  and  $\mathbf{x}_B$ . The inhomogeneity  $V(x, y, z)$  is located between depths  $z_m$  and  $z_M$ , and the space beyond  $z_M + \epsilon$  is assumed homogeneous. A receiver located at the same depth as the source with coordinates  $\mathbf{x}$  would record the reflection response  $R(\mathbf{x}, \mathbf{x}_A, \omega)$ , while a receiver located deeper than  $z_M$  would record the transmission response  $T(\mathbf{x}, \mathbf{x}_A, \omega)$ . To obtain a relationship between the reflection and transmission responses  $R$  and  $T$ , the one-way reciprocity theorem of the correlation type, equation 1, is employed. With states  $A$  and  $B$  defined as above, we have the following at depth  $z_m$ ,

$$P_{A,B}^+(\mathbf{x}, \mathbf{x}_{A,B}, \omega) = \delta(x - x_{A,B}) \delta(y - y_{A,B}) s_{A,B}(\omega) \quad (2)$$

and

$$P_{A,B}^-(\mathbf{x}, \mathbf{x}_{A,B}, \omega) = R(\mathbf{x}, \mathbf{x}_{A,B}, \omega) s_{A,B}(\omega), \quad (3)$$

where  $s$  represents the source signature. At depth  $z_M$ ,

$$P_{A,B}^+(\mathbf{x}, \mathbf{x}_{A,B}, \omega) = T(\mathbf{x}, \mathbf{x}_{A,B}, \omega) s_{A,B}(\omega) \quad (4)$$

and

$$P_{A,B}^-(\mathbf{x}, \mathbf{x}_{A,B}, \omega) = 0. \quad (5)$$

Substituting these into equation 1 and dividing by  $s_A^*(\omega) s_B(\omega)$ , we find

$$\int_{z_M} d^2\mathbf{x} T^*(\mathbf{x}, \mathbf{x}_A, \omega) T(\mathbf{x}, \mathbf{x}_B, \omega) + \int_{z_m} d^2\mathbf{x} R^*(\mathbf{x}, \mathbf{x}_A, \omega) R(\mathbf{x}, \mathbf{x}_B, \omega) = \delta(\mathbf{x}_B - \mathbf{x}_A). \quad (6)$$

A similar equation can be written for the elastic case (see Wapenaar et al., 2004). This relation provides amplitude information about the transmission response from recorded reflection data and vice versa. However, all phase information is lost in this process, and there is no unique way to recover it from this relation alone. Reconstructing the phase from the amplitude for a signal would require additional information, which is sometimes provided by assuming the minimum-phase condition. For 1D acoustic media, Herman (1992) and Wapenaar and Herrmann (1993) describe the procedure of constructing the transmission response from reflection data. The procedure implicitly uses the fact that the transmission response for this type of medium is minimum phase, so its full phase can be reconstructed from amplitude information only. This property of the recorded wavefield depends on the medium that the wave propagates through; in general it is not satisfied. In other words, even when the source wavelet used in a seismic experiment is minimum phase, the interaction with complex subsurface structures can result in a nonminimum-phase signal being recorded on the measurement surface. Dif-

ferent descriptions of minimum-phase signals in the time and frequency domains, as well as properties of minimum-phase media and reflection coefficients, have been collected by Nita and Weglein (2005). Bostock (2004) points out some of the cases in which the wavefield preserves the minimum-phase property, namely for pre-critical intramodal free-surface reverberations and transmitted P-waves in weak- to moderate-contrast stratification with small horizontal wavenumbers. However, for general acoustic and elastic media and wavefield propagation with arbitrary horizontal wavenumber, the recorded signal will have a mixed-phase character.

In this paper, we present a new approach for obtaining data at depth from measured reflection data based on the inverse scattering series that is fundamentally different from the daylight imaging approach. Our objective, having provided a brief overview of the current thinking behind reflection-to-transmission data prediction, is to present new, embryonic ideas concerning potential contributions of the inverse-scattering series to the problem of constructing data that were not acquired from data that were acquired and predicting the data both beneath and within the target medium. This involves two different approaches and extensions of previous ideas. The first is a linear reflection-to-transmission map (Stolt, 2002), and the second is a construction of the wavefield at depth without the velocity model (Weglein et al., 2001). The exposition is somewhat involved, so we include a table of symbols (Table 1) for reference.

The former linear extrapolation and interpolation method derives from an exact linear relationship within the inverse-scattering series and equally accommodates primaries and multiples without requiring a velocity model for either type of event. The cost in the inverse series depth imaging approach is that a suite of frequencies from the surface data is required to downward continue a single temporal frequency of the wavefield when the medium is unknown<sup>5</sup>.

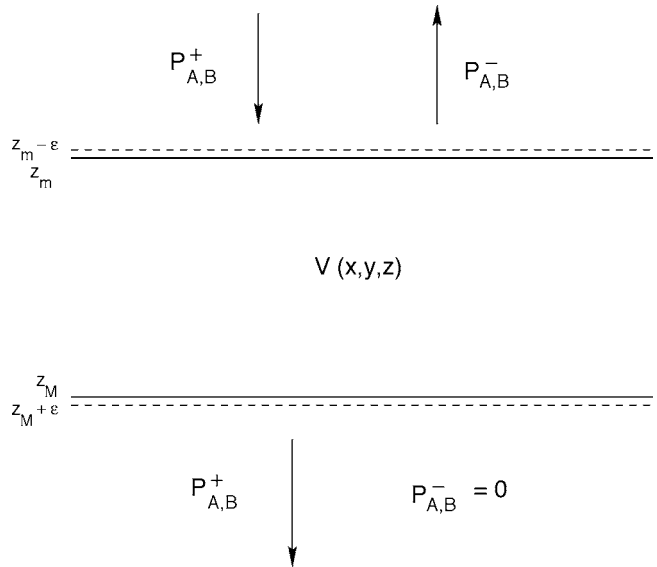


Figure 1. The choice of acoustic states in an experiment without free surface and corresponding up going and down going wavefields.

<sup>5</sup>When the medium is known, one temporal frequency of surface data determines the same temporal frequency of the wavefield at depth (all current methods for constructing the wavefield at depth fall into this category and can be viewed as following from Green's theorem, i.e., they are linear in the surface data). When the medium is unknown, the inverse-scattering series prescribes a nonlinear combination of surface data at a suite of frequencies to predict one given frequency of the wavefield at depth. These concepts and insights are the direct extension of lessons learned about temporal frequency requirements and linear/nonlinear processes in the removal of free-surface and internal multiples [see the tutorial for chapters 4 and 5 in Weglein and Dragoset (2005)].

Table 1. Nomenclature

Symbols	
$\mathbb{1}$	Unit operator
$c, c_0$	Actual, reference velocity
$\mathbf{G}, \mathbf{G}_0$	Actual, reference Green's operator
$\mathbf{k} = (\omega/c_0)\hat{\mathbf{k}}$	Incident wave vector
$\mathbf{k}' =  \mathbf{k} \hat{\mathbf{f}}$	A wave vector directed toward observation point $\hat{\mathbf{f}}$
$\hat{\mathbf{k}}$	Unit vector in the direction of $\mathbf{k}$
$(x_g, y_g, z_g)$	Receiver coordinates
$(x_s, y_s, z_s)$	Source coordinates
$k_{g_x}, k_{g_y}$	Fourier conjugates to $x_g, y_g$
$k_{s_x}, k_{s_y}$	Fourier conjugate to $x_s, y_s$
$k_{m_x}, k_{m_y}$	Midpoint-conjugate variables
$k_{h_x}, k_{h_y}$	Offset-conjugate variables
$\mathbf{L}, \mathbf{L}_0$	Actual, reference differential operator
$\mathbf{P}, \mathbf{P}_0$	Actual, reference wave operator
$\mathbf{p} = (p_x, p_y, p_z)$	3D Fourier conjugate to $\mathbf{r} = (x, y, z)$
$p_{g_x}, p_{g_y}, p_{g_z}, p_{s_x}, p_{s_y}, p_{s_z}$	Fourier conjugates to $x_g, x_s, y_g, y_s$
$p_{g_z}, p_{s_z}$	Fourier conjugates to $z_g, z_s$
$P^+, P^-$	Flux-normalized down going, up going wavefields
$q_g, q_s$	Depth wavenumbers
$R$	Reflection response
$\mathbf{r}, \mathbf{r}', \mathbf{r}_s$	Field point, scattering point, source point
$s, s_{A,B}, s_A, s_B, S(\omega)$	Source signature
$T$	Transmission response
$\mathbf{T}(\mathbf{k}, \mathbf{k}'), \mathbf{T}(\mathbf{p}, \mathbf{p}')$	T-matrix, generalized T-matrix
$\mathbf{V}$	Perturbation operator
$V(x, y, z)$	Inhomogeneity located between $z_m$ and $z_M$
$W(\mathbf{k})$	Fourier transform of $\alpha(\mathbf{r})$
$\mathbf{x}_A, \mathbf{x}_B$	Spatial coordinates of the sources
$z_0$	The location of the delta scatter
$z_M, z_m$	Upper, lower limit of inhomogeneity $V(x, y, z)$
$\alpha(\mathbf{r})$	Velocity perturbation
$\alpha_1^R$	Reflection-derived linear inverse
$\alpha_1^T$	Transmission-derived linear inverse
$\omega$	Temporal frequency
$\Psi_s$	Scattered field operator
$\psi_s, \phi_s$	Scattered wavefields
$\psi_s^{(n)} n = 1, 2, \dots$	Portion of the wavefield that is $n$ th order in the measured data
$\lambda$	Amplitude of the delta scatter

## AN INVERSE-SCATTERING SERIES FRAMEWORK FOR CONSTRUCTING THE WAVEFIELD AT DEPTH AND THE TRANSMISSION RESPONSE FROM REFLECTION DATA

In this section, we describe the opportunity provided by the inverse-scattering series for constructing, from reflection data, the actual wavefield at all depths, including the transmitted wavefield. The latter is achievable without requiring or determining the earth material properties needed by linear Green's theorem-based approaches. Early thinking on this approach for determining the wavefield at depth from the inverse-scattering series was first described by Weglein et al. (2000). To describe this procedure, we first introduce a data format known in the literature of scattering theory (e.g., Goldberger and Watson, 1964; Taylor, 1972) as the T-matrix and the experiment that motivated that definition. We then describe the generalization of that experiment-motivated T-matrix definition and how that generalized T-matrix played a role in early inverse-scattering series papers of Moses (1956), Razavy (1975), and Weglein et al. (1981). The relationship between seismic recorded data and a T-matrix format was presented by Weglein et al. (1981) and Stolt and Jacobs (1981). We then describe the challenge within the inverse-scattering series, that the presence of the full generalized T-matrix is represented, and note how those papers address that issue. In addressing that requirement for the complete T-matrix resides the kernel of a wavefield-at-depth concept and methodology. In fact, we show that satisfying the full off-shell T-matrix requirement, order-by-order in the measured data, as described by the classic inverse-scattering series papers of Moses and Razavy cited above, is equivalent to constructing the actual scattered, and total, wavefields at depth. This formalism can accommodate acoustic, elastic, and inelastic media, requiring only knowledge of the source signature in the water as input, but without assumptions on the phase of the wavelet.

### Background

Scattering theory is a form of perturbation analysis, in which a perturbation in the properties of a medium is related to a perturbation in a wavefield that experiences that medium. The original, unperturbed medium is typically labeled as the reference medium. The difference between the actual and reference media is characterized by the perturbation operator. The corresponding difference between the actual and reference wavefields is called the scattered wavefield. Forward scattering takes as input the reference medium, the reference wavefield, and the perturbation operator and then outputs the actual wavefield. Inverse scattering takes as input the reference medium, the reference wavefield, and values of the actual field on the measurement surface and outputs the difference between actual and reference medium properties through the perturbation operator. Inverse-scattering theory methods typically assume the support of the perturbation to be on one side of the measurement surface. In seismic application, this condition translates to a requirement that the difference between actual and reference media be nonzero only below the source-receiver surface. Conversely, in seismic applications, inverse-scattering methods require that the reference medium agree with the actual medium at and above the measurement surface.

For the marine seismic application, the sources and receivers are located within the water column and the simplest reference medium (that satisfies the above-stated conditions) is a half-space of water bounded by a free surface at the air-water interface. Because scatter-

ing theory relates the difference between actual and reference wave-fields to the difference between their medium properties, it is reasonable that the mathematical description begin with the differential equations governing wave propagation in these media. Let

$$\mathbf{L}G = -\delta(\mathbf{r} - \mathbf{r}_s) \quad (7)$$

and

$$\mathbf{L}_0G_0 = -\delta(\mathbf{r} - \mathbf{r}_s), \quad (8)$$

where  $\mathbf{L}$ ,  $\mathbf{L}_0$  and  $G$ ,  $G_0$  are the actual and reference differential operators and Green's functions, respectively, for a single temporal frequency  $\omega$ ,  $\delta(\mathbf{r} - \mathbf{r}_s)$  is the Dirac delta function, and  $\mathbf{r}$  and  $\mathbf{r}_s$  are the field point and source location, respectively. Equations 7 and 8 assume that the actual source and receiver signatures have been deconvolved. The quantities  $G$  and  $G_0$  are the matrix elements of the Green's operators,  $\mathbf{G}$  and  $\mathbf{G}_0$ , in the spatial coordinates and the temporal frequency domain;  $\mathbf{G}$  and  $\mathbf{G}_0$  themselves satisfy  $\mathbf{L}\mathbf{G} = -\mathbf{1}$  and  $\mathbf{L}_0\mathbf{G}_0 = -\mathbf{1}$ , where  $\mathbf{1}$  is the unit operator. A further set of operators based on differences between these quantities are considered next:

$$\mathbf{V} \equiv \mathbf{L} - \mathbf{L}_0 \quad (9)$$

and

$$\Psi_s \equiv \mathbf{G} - \mathbf{G}_0; \quad (10)$$

$\mathbf{V}$  is the perturbation operator and  $\Psi_s$  is the scattered field operator. The Lippmann-Schwinger equation is the fundamental equation of scattering theory, an operator identity that relates  $\Psi_s$ ,  $\mathbf{G}_0$ ,  $\mathbf{V}$ , and  $\mathbf{G}$  (Taylor, 1972):

$$\Psi_s = \mathbf{G} - \mathbf{G}_0 = \mathbf{G}_0\mathbf{V}\mathbf{G}. \quad (11)$$

When equations 7 and 8 include a wavelet, we reexpress equation 11 as

$$\mathbf{P}_s = S(\omega)\Psi_s = \mathbf{P} - \mathbf{P}_0 = \mathbf{G}_0\mathbf{V}\mathbf{P}, \quad (12)$$

where  $\mathbf{P}_0$ ,  $\mathbf{P}$  are the reference and actual wave operators respectively, and  $\mathbf{G}_0$  is the causal reference Green's operator.

In the coordinate representation, equation 12, alternatively called the scattering equation, is valid for all positions of  $\mathbf{r}$  and  $\mathbf{r}_s$  whether or not they are inside the support of  $\mathbf{V}$ . Examples of  $\mathbf{L}$ ,  $\mathbf{L}_0$ , and  $\mathbf{V}$  (when  $\mathbf{P}$  corresponds to a pressure field) in an inhomogeneous, variable velocity constant density medium, are:

$$L = \nabla^2 + \frac{\omega^2}{c^2(\mathbf{r})},$$

$$L_0 = \nabla^2 + \frac{\omega^2}{c_0^2},$$

and

$$V = L - L_0 = -k^2\alpha(\mathbf{r}), \quad (13)$$

where  $k = \omega/c_0$  and  $\alpha = 1 - c_0^2/c^2$ . For this model, equation 12 becomes,

$$\begin{aligned} P(\mathbf{r}, \mathbf{r}_s, \omega) &= P_0(\mathbf{r}, \mathbf{r}_s, \omega) \\ &- \int G_0(\mathbf{r}, \mathbf{r}', \omega) k^2 \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{r}_s, \omega) d\mathbf{r}' \\ &= P_0(\mathbf{r}, \mathbf{r}_s, \omega) \\ &- \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} k^2 \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{r}_s, \omega) d\mathbf{r}'. \end{aligned} \quad (14)$$

We next proceed, using the above form as a starting point, to review and develop some of the key quantities used in the scattering description and formulation.

### The T-matrix and the seismic wavefield

Let us consider the special case where a single frequency plane wave,

$$\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{(2\pi)^{3/2}}, \quad (15)$$

is incident upon a localized target  $\alpha(\mathbf{r})$ , in which the support of  $\alpha$  is within a sphere of radius  $R$  and  $\mathbf{k} = |\mathbf{k}|\hat{\mathbf{k}} = \omega/c_0\hat{\mathbf{k}}$ . Consider measuring the wavefield at a position  $\mathbf{r}$  in the far-field well beyond the support of  $\alpha$ . For that experiment, the scattering equation becomes

$$P(\mathbf{r}, \mathbf{k}) = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{(2\pi)^{3/2}} - \frac{e^{ik|\mathbf{r}|}}{4\pi|\mathbf{r}|} \int e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} k^2 \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{k}) d\mathbf{r}', \quad (16)$$

where we have substituted the far-field form of the Green's function.

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \approx \frac{1}{4\pi|\mathbf{r}|} e^{ik\hat{\mathbf{r}}\cdot\mathbf{r}'}, \quad (17)$$

in propagating from the scattering point  $\mathbf{r}'$  to the field point  $\mathbf{r}$ ;  $\hat{\mathbf{r}}$  is the unit vector in the direction of  $\mathbf{r}$ . In the far-field,  $P_s$ , from equation 12 (with  $\mathbf{k}' \equiv k\hat{\mathbf{r}}$ ), becomes

$$P_s(\mathbf{r}, \mathbf{k}) \approx - \frac{e^{ik|\mathbf{r}|}}{4\pi|\mathbf{r}|} \int e^{-ik'\cdot\mathbf{r}'} k^2 \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{k}) d\mathbf{r}'. \quad (18)$$

This is a spherical wave with an amplitude of

$$\int e^{-ik'\cdot\mathbf{r}'} k^2 \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{k}) d\mathbf{r}' \quad (19)$$

that depends on  $\alpha(\mathbf{r})$ , the incident wave vector  $\mathbf{k}$ , and a vector  $\mathbf{k}'$  of the same magnitude pointing in the direction of the observation point  $\mathbf{r}$ . The message of equation 18 is that in the far-field, the scattered wave is a spherical wave with an angle-dependent amplitude that captures the properties of the actual (in general) extended target  $\alpha(\mathbf{r})$ .

The above observation motivates the definition of what is referred to as the T-matrix ('T' for transition; e.g., Taylor, 1972), a quantity relating the strength of the far-field scattered wave in direction  $\hat{\mathbf{r}} = \mathbf{k}'/|\mathbf{k}'|$  resulting from the incident plane wave with the wave vector  $\mathbf{k}$ :

$$T(\mathbf{k}, \mathbf{k}') \equiv k^2 \int \frac{e^{-i\mathbf{k}' \cdot \mathbf{r}'}}{(2\pi)^{3/2}} \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{k}) d\mathbf{r}'. \quad (20)$$

In the experiment we use to motivate this definition,  $\mathbf{k}$  and  $\mathbf{k}'$  have the same magnitude  $\omega/c_0$ . For the purposes of inverse-scattering theory, it is useful to generalize the T-matrix definition to

$$T(\mathbf{p}, \mathbf{p}') \equiv \int \frac{e^{-i\mathbf{p}' \cdot \mathbf{r}'}}{(2\pi)^{3/2}} k^2 \alpha(\mathbf{r}') P(\mathbf{r}', \mathbf{p}) d\mathbf{r}', \quad (21)$$

where  $\mathbf{p}$  and  $\mathbf{p}'$  are two arbitrary vectors, unrelated to  $\mathbf{k}$  in either magnitude or direction.

The data format represented by equation 20 is the standard found in the papers by Moses (1956) and Razavy (1975). It can, for instance, be interpreted for  $\mathbf{k}' = \mathbf{k}$  and  $\mathbf{k}' = -\mathbf{k}$  as the forward scattering amplitude and backscattering amplitude, respectively. The relationship between T-matrix incident plane wave, far-field measurements (equation 20), and reflection seismic data was first described by Weglein et al. (1981) and further analyzed and developed by Stolt and Jacobs (1981). In three dimensions we have

$$\begin{aligned} \frac{e^{i(q_g z_g + q_s z_s)} T(\mathbf{k}'; \mathbf{k})}{q_g q_s} &= \frac{e^{i(q_g z_g + q_s z_s)} T(k_{gx}, k_{gy}, -q_g; k_{sx}, k_{sy}, q_s)}{q_g q_s} \\ &= \tilde{\phi}_s(k_{gx}, k_{gy}, z_g; k_{sx}, k_{sy}, z_s; \omega), \end{aligned} \quad (22)$$

where  $k_{gx}, k_{gy}$  and  $k_{sx}, k_{sy}$  are the Fourier conjugates to  $x_g, y_g$  and  $x_s, y_s$ , the depth wavenumbers  $q_g$  and  $q_s$  are

$$q_g = \text{sgn}(\omega) \left[ \left( \frac{\omega}{c_0} \right)^2 - k_{gx}^2 - k_{gy}^2 \right]^{1/2}, \quad (23)$$

$$q_s = \text{sgn}(\omega) \left[ \left( \frac{\omega}{c_0} \right)^2 - k_{sx}^2 - k_{sy}^2 \right]^{1/2},$$

and  $z_g, z_s$  are, as earlier, the depths of the receiver and source, respectively. The value  $\tilde{\phi}_s$  is the seismic scattered (reflection) data from a point source and measured at a point receiver. Beyond this, Weglein et al. (2003) have shown that the T-matrix expression  $T(\mathbf{p}, \mathbf{k})$ , where  $\mathbf{k} = \omega/c_0 \hat{\mathbf{k}} = (k_x, k_y, q_s)$  and  $\mathbf{p}$  is an arbitrary three-dimensional vector unrelated to  $\mathbf{k}$ , can also be related to the seismic exploration experiment. For arbitrary  $\mathbf{p}$ , the 3D relationship between  $\tilde{\phi}(\mathbf{p}, \mathbf{k})$  and  $T(\mathbf{p}, \mathbf{k})$  is

$$\begin{aligned} \tilde{\phi}_s(p_x, p_y, p_z; k_x, k_y, q_s) &= \tilde{\phi}_s(\mathbf{p}; k_x, k_y, q_s) \\ &= \frac{T(\mathbf{p}, \mathbf{k})}{k^2 - p^2 - i\varepsilon} \left( \frac{1}{q_s} \right), \end{aligned} \quad (24)$$

where  $\mathbf{p}$  is the 3D Fourier conjugate of the wavefield observation vector  $\mathbf{r} = (x, y, z)$ , which is not confined to the measurement surface  $z = z_g$ ; further, the magnitude of  $\mathbf{p}$  is unrelated to the magnitude or direction of  $\mathbf{k}$ . The interpretation that follows from equation 24 is that  $T(\mathbf{p}, \mathbf{k})$  with  $\mathbf{k} = \omega/c_0 \hat{\mathbf{k}}$  and  $\mathbf{p}$  arbitrary is equivalent to knowing the scattered field at all spatial locations (i.e., all  $x, y$  and all depths  $z$ ) for a suite of single frequency experiments, at frequency  $\omega$ , with sources located at  $(x_s, y_s, z_s)$ . The variable  $\mathbf{p}$  is the 3D conjugate of  $(x, y, z)$  in all 3D space, and  $\mathbf{k}$  comes from a Fourier transform over

single-frequency point sources on the surface. The inverse-scattering series solution for  $\alpha(\mathbf{r})$  is described in equations 20–22 of Weglein et al. (1981); further, the reader may wish to examine equations 32–40 of Weglein et al. (2003) for further discussion on what we next review.

To generate the inverse-scattering series for the constant density variable velocity acoustic case, we first define the Fourier transform of  $\alpha(\mathbf{r})$  as

$$W(\mathbf{k}) \equiv \int e^{2i\mathbf{k} \cdot \mathbf{r}'} \frac{\alpha(\mathbf{r}')}{(2\pi)^{3/2}} d\mathbf{r}'. \quad (25)$$

To determine  $W(\mathbf{k})$  for all  $\mathbf{k}$  is to determine the inverse solution for  $\alpha(\mathbf{r})$ . Equation 21 of Weglein et al. (1981) then relates  $W(\mathbf{k})$  to the T-matrix quantities as

$$\begin{aligned} W(\mathbf{k}) &= - \left( \frac{2}{\pi} \right)^{1/2} \frac{b(\mathbf{k})}{k^2} + k^2 \int T^*(\mathbf{k}, \mathbf{q}) \mathcal{T}(-\mathbf{k}, \mathbf{q}) \\ &\times \left\{ \frac{H(k)}{q^2 - k^2 - i\varepsilon} + \frac{H(-k)}{q^2 - k^2 + i\varepsilon} \right\} d\mathbf{q}, \end{aligned} \quad (26)$$

where

$$\begin{aligned} b(\mathbf{k}) &= \frac{k^2 \mathcal{T}(-\mathbf{k}, \mathbf{k})}{\left( \frac{2}{\pi} \right)^{1/2}} \\ &= \tilde{\phi}_s(-k_x, -k_y, -q_s; k_x, k_y, q_s) q_s^2 e^{-iq_s(z_s + z_g)} \end{aligned} \quad (27)$$

and where  $k^2 \mathcal{T} \equiv T$  and  $H(k)$  is the Heaviside step function. Note that the second term on the right-hand side of equation 26 requires  $T(\mathbf{k}, \mathbf{q})$  for all  $\mathbf{q}$  for a given  $\mathbf{k} = \omega/c_0 \hat{\mathbf{k}}$ ,  $\mathbf{k} = (k_x, k_y, q_s)$ , where  $\mathbf{q}$  is an arbitrary 3D vector per its whole-space integration,  $\int d\mathbf{q}$ .

Therefore, the inverse-scattering series solution in the Fourier domain can be interpreted as requiring the scattered field (and, thus, the entire wavefield) at all points in space, not only on the measurement surface. Included in that requirement on the right-hand side of equation 26 is  $T(\mathbf{q}, \mathbf{k})$  when  $\mathbf{q} = \mathbf{k}$ , or rather  $T(\mathbf{k}, \mathbf{k})$ . The latter is the transmitted component of the scattered field.

Weglein et al. (2003) review the procedure first developed by Moses (1956) to address this wavefield-everywhere requirement. The inverse-scattering series produces all quantities associated with an unknown subsurface as a series order-by-order in the measured surface reflection data. The scattered (or total) wavefield at depth is no exception. In fact, the wavefield at depth is explicitly calculated, order-by-order, at each step within the calculation of  $W(\mathbf{k})$ , the Fourier transform of  $\alpha(\mathbf{r})$ ,

$$W(\mathbf{k}) = W_1(\mathbf{k}) + W_2(\mathbf{k}) + W_3(\mathbf{k}) + \dots, \quad (28)$$

and hence, through the inverse Fourier transform of  $W$ , the calculation of  $\alpha(\mathbf{r})$ . The relevant equation for wavefield construction is

$$T(\mathbf{p}, \mathbf{k}) = T_1(\mathbf{p}, \mathbf{k}) + T_2(\mathbf{p}, \mathbf{k}) + T_3(\mathbf{p}, \mathbf{k}) + \dots, \quad (29)$$

where  $T(\mathbf{p}, \mathbf{k})$  is

$$T(p_x, p_y, p_z; k_x, k_y, q_s).$$

Weglein et al. (2000) demonstrate that the first-order approximation to  $T(\mathbf{p}, \mathbf{k})$ , i.e.,  $T_1(\mathbf{p}, \mathbf{k})$ , or  $\tilde{\phi}_s$ , at all depths requires all frequency components contained within the surface data to predict one fre-

quency of  $\tilde{\phi}_s$  at all depths. Hence, the inverse-scattering series allows one frequency of the wavefield at all depths in the earth to be predicted as a series, order-by-order in the surface data, using all the frequency information therein. We describe this prediction in detail in the following section. That prediction of the scattered field at depth does not involve a back-propagation within the medium at depth; if the medium at depth is known, then one frequency on the surface predicts the scattered wavefield at depth at that frequency. This is a direct analog of the inverse-scattering internal multiple attenuator (Weglein et al., 1997) where, absent of any subsurface information, a suite of frequencies of surface data are required to predict one frequency of the output.

The latter one-frequency-in, one-frequency-out wavefield prediction, when the medium is known, derives from Green's theorem with Dirichlet, Neumann, or Robin boundary conditions and is the theory underlying all current migration theory. The inverse-scattering series predicts the wavefield at depth, including the transmitted wavefield, without knowing the medium, but all frequencies in the measured wavefield are required to produce a series solution for the wavefield at depth for any given frequency. The source signature in the reference medium is a prerequisite for this and all inverse scattering series applications. These concepts can be applied to all models that allow a perturbative expression. Among the models accommodated are acoustic, elastic, and inelastic media. However, it appears that the specific details of the wavefield construction will depend on the assumed model type. Transmission data can mitigate that model-type dependence for a wavefield at depth construction, whose purpose is to determine an accurate structural map.

### ORDER-BY-ORDER COMPUTATION OF THE WAVEFIELD AT DEPTH

The inverse-scattering series, considered from the standpoint of the T-matrix, constructs the wavefield everywhere in the unknown medium, order-by-order, in the scattered wavefield measured outside of that medium.

When the predicted wavefield is combined with a causality-based, small time imaging condition, its output is a reflectivity function map. Such an output quantity would correspond to an angle-dependent reflection coefficient, or scattering operator, for a specular or nonspecular target, respectively (e.g., Weglein and Stolt, 1999).

The approach begins with the linear component of the inverse-scattering series, in which  $D = G_0 V_1 G_0$ , and explicitly solves for  $V_1$  from the data for a given earth model type. For a constant-density variable-velocity acoustic model, the perturbation  $V$  can be written, as in equation 13, as  $V = -k^2 \alpha(x, y, z)$ , where  $k = \omega/c_0$ ,  $c_0$  is the constant reference velocity and  $\alpha(x, y, z)$  is the variation of the index of refraction. The linear relationship becomes

$$\alpha_1(p_{g_x} - p_{s_x}, p_{g_y} - p_{s_y}, -q_g - q_s) = \frac{4q_g q_s}{-k^2} \psi_s(p_{g_x}, p_{g_y}, z_g, p_{s_x}, p_{s_y}, z_s; \omega), \quad (30)$$

where  $\psi_s(p_{g_x}, p_{g_y}, z_g, p_{s_x}, p_{s_y}, z_s; \omega)$  is the measured scattered field, i.e., the data  $D$ , from a set of experiments with sources at  $(x_s, y_s, z_s)$  and receivers at  $(x_g, y_g, z_g)$ . The quantities  $z_s, z_g$  are the constant depth of source and receiver, respectively;  $p_{g_x}, p_{s_x}, p_{g_y},$  and  $p_{s_y}$  are the Fourier conjugates of  $x_g, x_s, y_g$  and  $y_s$ , and the vertical wavenumbers are defined by

$$q_g = \text{sgn}(\omega) \left[ \left( \frac{\omega}{c_0} \right)^2 - p_{g_x}^2 - p_{g_y}^2 \right]^{1/2}$$

and

$$q_s = \text{sgn}(\omega) \left[ \left( \frac{\omega}{c_0} \right)^2 - p_{s_x}^2 - p_{s_y}^2 \right]^{1/2}. \quad (31)$$

In contrast with equation 30, the first-order wavefield at depth is

$$\begin{aligned} \psi_s^{(1)}(p_{g_x}, p_{g_y}, p_{g_z}, p_{s_x}, p_{s_y}, p_{s_z}; \omega) \\ = \frac{k^2 \alpha_1(p_{g_x} - p_{s_x}, p_{g_y} - p_{s_y}, -p_{g_z} - p_{s_z})}{(p_{s_x}^2 + p_{s_y}^2 + p_{s_z}^2 - k^2 - i\epsilon)(p_{g_x}^2 + p_{g_y}^2 + p_{g_z}^2 - k^2 - i\epsilon)}, \end{aligned} \quad (32)$$

where  $(p_{s_x}, p_{s_y}, p_{s_z}), (p_{g_x}, p_{g_y}, p_{g_z})$  are the conjugate variables to  $(x_s, y_s, z_s)$  and  $(x_g, y_g, z_g)$ , respectively, and  $\epsilon$  is a small positive parameter needed to ensure causal Green's functions. Equation 32 follows from the Lippmann-Schwinger equation by expanding each of  $\psi_s$  (at depth),  $V$ , and  $G$  in orders of the surface data  $(\psi_s)_m = D$  and then equating terms of equal order in  $D$  on both sides of the equation. The scattered wavefield  $\psi_s$ , in general, for any source and receiver position may be expressed as

$$\psi_s = G_0 k^2 \alpha \psi, \quad (33)$$

and  $\alpha$  and  $\psi$  are expandable in terms of the data  $(\psi_s)_m = D$ :

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \dots, \quad (34)$$

$$\psi = \psi_0 + \psi_1 + \psi_2 + \psi_3 + \dots,$$

where

$$\psi_0 = G_0,$$

$$\psi_s^{(1)} = \psi_1 = G_0 k^2 \alpha_1 G_0, \quad (35)$$

$$\psi_s^{(2)} = \psi_2 = G_0 k^2 \alpha_2 G_0 + G_0 k^2 \alpha_1 (\psi_1),$$

etc. These equations are valid at all spatial locations, including in the medium, and they form the basis of the velocity-independent wavefield construction method. We point out that these equations are not the inverse-scattering series equations. The latter are a relationship between the measured values of the wavefield and an order-by-order construction of  $\alpha$ . Equation 35 represents something entirely different, an order-by-order construction of the wavefield  $\psi_s$  at depth. The procedure starts with  $\alpha_1$  determined by  $(\psi_s)_m$ , the measured values of  $\psi_s$ , followed by the substitution of  $\alpha_1$  into the expression for  $\psi_s^{(1)}$ . Then  $\alpha_1, \alpha_2$ , and  $\psi_1$  are used to construct  $\psi_s^{(2)}$ , the second-order approximation of the wavefield at depth. For sources and receivers at depth,

$$\psi_s = \psi_s^{(1)} + \psi_s^{(2)} + \dots, \quad (36)$$

$$G = G_0 + \psi_s,$$

where  $\psi_s^{(n)}(x_g, y_g, z_g, x_s, y_s, z_s; \omega)$  is the portion of the wavefield at depth that is  $n$ th order in the measured data. Within equation 35, there is the opportunity to include only task-separated portions of  $\alpha_2$  rather than its entirety. The inclusion of only reflector location terms, or imaging terms within  $\alpha_2$  (as in Shaw et al., 2004), might be expected to produce a structurally accurate imaged wavefield without computing the complete  $\alpha_2$ . Whether the complete  $\alpha_2$ , or a portion thereof, is computed and input into equation 35 as part of the construction of  $\psi_s^{(2)}$ , this predicted wavefield at depth (to second order) is never back-propagated through an actual, or updated, medium. All back-propagation occurs in the original, unchanged reference medium. On the measurement surface [throughout this paper, the form  $(\cdot)_m$  indicate the quantity  $(\cdot)$  evaluated on a measurement surface], these terms reduce to

$$(\psi_s)_m = (\psi_s^{(1)})_m, \quad (37)$$

$$(\psi_s^{(2)})_m = (\psi_s^{(3)})_m = \dots = 0.$$

In detail, the first-order field at depth in terms of the wavefield on the measurement surface is

$$\begin{aligned} & \psi_s^{(1)}(p_{g_x}, p_{g_y}, p_{g_z}, p_{s_x}, p_{s_y}, p_{s_z}; \omega) \\ & - 4 \frac{k^2 q'_g q'_s}{k_1^2} [\psi_s(p_{g_x}, p_{g_y}, q'_g, p_{s_x}, p_{s_y}, q'_s; \omega_1)]_m \\ & = \frac{\quad}{(p_{s_x}^2 + p_{s_y}^2 + p_{s_z}^2 - k^2 - i\epsilon)(p_{g_x}^2 + p_{g_y}^2 + p_{g_z}^2 - k^2 - i\epsilon)}, \end{aligned} \quad (38)$$

where

$$k_1 = \frac{\omega_1}{c_0}, \quad (39)$$

$$q'_{g,s} = \text{sgn}(\omega) \left[ \frac{\omega_1^2}{c_0^2} - p_{g_x, s_x}^2 - p_{g_y, s_y}^2 \right]^{1/2},$$

and

$$\begin{aligned} \left( \frac{\omega_1}{c_0} \right)^2 & = p_{s_x}^2 + p_{s_y}^2 \\ & + \left[ \frac{p_{s_x}^2 + p_{s_y}^2 - p_{g_x}^2 - p_{g_y}^2 - (p_{s_z} - p_{g_z})^2}{2(p_{s_z} - p_{g_z})} \right]^2. \end{aligned} \quad (40)$$

The quantity  $\psi_s^{(1)}(x_g, y_g, z_g, x_s, y_s, z_s; \omega)$  for any  $(x_s, z_s)$  and  $(x_g, z_g)$  follows by inverse Fourier transforming equation 38. To find the first-order estimate of the wavefield at all depths for a single frequency  $\omega$  requires sweeping through surface data for all frequencies  $\omega$  in order to fill the spectrum of the source and receiver depth variable's Fourier conjugates,  $p_{s_x}$  and  $p_{g_x}$ , respectively, for the left-hand side of equation 38. That calculation provides the first-order wavefield at depth

without the velocity. The higher-order computations of the wavefield at depth without the velocity follow directly from equations 35.

## ANALYSIS OF A LINEAR REFLECTION-TO-TRANSMISSION MAPPING

In this section we consider an approximation to the reflection-to-transmission problem motivated by the order-by-order construction of the transmitted wavefield that is a natural by-product of the full inverse-scattering series. We also consider a linear-linear mapping of reflected to transmitted wavefield data and comment on its potential level of applicability and accuracy.

The fact that the first equation in the inverse-scattering series,  $D = G_0 k^2 \alpha_1 G_0$  (Weglein et al., 2003), solves for a factor  $\alpha_1$  that treats all events on equal footing (whether they are primaries or multiples) is a deficit from an inversion point of view but a definite asset from a data reconstruction point of view and objective. For the purpose of data reconstruction, we suggest that the idea of (1) seeking to construct a (linear) earth model and then (2) using that earth model to reconstruct data is overly restrictive. Rather, we view  $\alpha_1$  as simply a factor in the equation that relates  $\alpha_1$  to  $D$  that is flexible enough to match the variability of the data. Adopting such a curve-fitting view provides a framework for the extrapolation of both primaries and multiples that does not include as input a knowledge of or determination of the actual medium velocity, while simultaneously maintaining a relatively simple linear form.

In this section we discuss a linear map of reflected-to-transmitted data. We first present a form of the mapping for a single-parameter 3D acoustic model, then examine it for a 1D normal-incidence problem and carry out an initial analysis regarding accuracy.

### A 3D single-parameter linear reflection-to-transmission mapping

We demonstrate the framework for a linear reflection-to-transmission mapping with a 3D, single-parameter scattering model that assumes medium wavespeed fluctuations away from a homogeneous fluid reference medium with constant density. Data  $D(x_g, y_g, z_g | x_s, y_s, z_s; \omega)$ , corresponding to the measurement of a scattered field on a surface defined by fixed  $z_g$  (receiver depth) and  $z_s$  (source depth), and variable lateral receiver and source locations  $x_g, y_g$  and  $x_s, y_s$  (respectively) is related to the linear component of the single-parameter acoustic scattering potential  $\alpha(x, y, z) = 1 - c_0^2/c^2(x, y, z)$ , namely,  $\alpha_1(x, y, z)$ , by

$$\begin{aligned} & D(k_{g_x}, k_{g_y}, z_g | k_{s_x}, k_{s_y}, z_s; \omega) \\ & = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' G_0(k_{g_x}, k_{g_y}, z_g | x', y', z'; \omega) \\ & \quad \times k^2 \alpha_1(x', y', z') G_0(x', y', z' | k_{s_x}, k_{s_y}, z_s; \omega), \end{aligned} \quad (41)$$

in which the lateral coordinates  $x_g, y_g$  and  $x_s, y_s$  have been Fourier-transformed into their conjugate-domain parameters  $k_{g_x}, k_{g_y}$  and  $k_{s_x}, k_{s_y}$ , respectively, and the  $G_0$ 's are 3D homogeneous acoustic Green's functions, given explicitly by

$$\begin{aligned}
G_0(k_{g_x}, k_{g_y}, z_g | x', y', z'; \omega) &= - \frac{e^{-ik_{g_x}x' - ik_{g_y}y'} e^{iq_g|z_g - z'|}}{2\pi i2q_g}, \\
G_0(x', y', z' | k_{s_x}, k_{s_y}, z_s; \omega) &= - \frac{e^{ik_{s_x}x' + ik_{s_y}y'} e^{iq_s|z' - z_s|}}{2\pi i2q_s}.
\end{aligned} \tag{42}$$

(The difference in sign on the source and receiver wavenumbers is from different sign conventions in the Fourier transform.) We next fix the source location to be at depth  $z'$  such that  $z' > z_s$  for all  $\alpha_1(x', z') \neq 0$ , which, in addition to the substitution of equations 42 into 41, leads to

$$\begin{aligned}
D(k_{g_x}, k_{g_y}, z_g | k_{s_x}, k_{s_y}, z_s; \omega) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \left[ \frac{e^{-ik_{g_x}x' - ik_{g_y}y'} e^{iq_g|z_g - z'|}}{i2q_g} \right] \\
&\quad \times k^2 \alpha_1(x', y', z') \left[ \frac{e^{ik_{s_x}x' + ik_{s_y}y'} e^{iq_s|z' - z_s|}}{i2q_s} \right] \\
&= - \frac{k^2}{16\pi^2 q_g q_s} e^{-iq_s z_s} \int_{-\infty}^{\infty} dz' e^{iq_g|z_g - z'|} e^{iq_s z'} \\
&\quad \times \alpha_1(k_{g_x} - k_{s_x}, k_{g_y} - k_{s_y}, z').
\end{aligned} \tag{43}$$

Data corresponding to a reflection-like geometry (called  $R$ ) and a transmission-like geometry ( $T$ ) derive from equation 43 simply by further fixing the receiver depths in each case to be, respectively, either smaller than all contributing  $z'$  values ( $z_g^R$ ) or greater than all contributing  $z'$  values ( $z_g^T$ ). Such data are then relatable, linearly, to portions of the scattering potential, let us say  $\alpha_1^R$  and  $\alpha_1^T$ , respectively. The inverse-scattering series could be solved for  $\alpha$  order-by-order in either reflection or transmission data.

In orders of reflected data  $\alpha = \alpha_1^R + \alpha_2^R + \dots$  and in orders of transmission data  $\alpha = \alpha_1^T + \alpha_2^T + \dots$ , the first term in each of these two series correspond to

$$\begin{aligned}
R(k_{g_x}, k_{g_y}, z_g^R | k_{s_x}, k_{s_y}, z_s; \omega) &= - \frac{k^2}{16\pi^2 q_g q_s} e^{-i(q_s z_s + q_g z_g^R)} \int_{-\infty}^{\infty} dz' e^{i(q_g + q_s)z'} \\
&\quad \times \alpha_1^R(k_{g_x} - k_{s_x}, k_{g_y} - k_{s_y}, z'), \\
T(k_{g_x}, k_{g_y}, z_g^T | k_{s_x}, k_{s_y}, z_s; \omega) &= - \frac{k^2}{16\pi^2 q_g q_s} e^{-i(q_s z_s - q_g z_g^T)} \int_{-\infty}^{\infty} dz' e^{i(q_s - q_g)z'} \\
&\quad \times \alpha_1^T(k_{g_x} - k_{s_x}, k_{g_y} - k_{s_y}, z').
\end{aligned} \tag{44}$$

Performing the Fourier transforms and solving for  $\alpha_1^R$ , we have

$$\begin{aligned}
\alpha_1^R(k_{g_x} - k_{s_x}, k_{g_y} - k_{s_y}, -q_g - q_s) &= - \frac{4q_g q_s}{k^2} e^{i(q_s z_s + q_g z_g^R)} R(k_{g_x}, k_{g_y}, z_g^R | k_{s_x}, k_{s_y}, z_s; \omega), \\
T(k_{g_x}, k_{g_y}, z_g^T | k_{s_x}, k_{s_y}, z_s; \omega) &= - \frac{k^2}{4q_g q_s} e^{-i(q_s z_s - q_g z_g^T)} \\
&\quad \times \alpha_1^T(k_{g_x} - k_{s_x}, k_{g_y} - k_{s_y}, q_g - q_s).
\end{aligned} \tag{45}$$

The idea here is to use the reflection-derived linear inverse  $\alpha_1^R$  as computed in equation 45 to estimate the transmitted data by substitution of  $\alpha_1^R$  for  $\alpha_1^T$ . It may be of particular interest to point out that this mapping is only possible in the case of a medium that has 2D or 3D variability. Notice that the linearized medium-transmitted wavefield relationship in equation 44 involves a Fourier transform over the depth  $z$ , in which the conjugate coordinate is the difference between source- and receiver-depth wavenumbers. In a 1D medium, these depth wavenumbers are equal, and as such the integral captures only the dc component of the vertical wavenumber of  $\alpha_1^T$ ; in order to fill the spectrum of  $\alpha_1^T$  with data information, we require a range of contributing  $q_g - q_s$  values, which, again, are only present if the medium has lateral variability.

If a source wavelet  $S(\omega)$  is included in the physical description, then the  $P_0$  and  $P$  are multiplied by  $S(\omega)$ , and  $G_0(x', y', z', k_{s_x}, k_{s_y}, z_s; \omega)$  in equation 41 is replaced by  $S(\omega)G_0(x', y', z', k_{s_x}, k_{s_y}, z_s; \omega)$  and equations 45 become

$$\begin{aligned}
S(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) \alpha_1^R(k_{m_x}, k_{m_y}, k_z) &= A^R(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) R(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z)
\end{aligned} \tag{46}$$

$$\begin{aligned}
S(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) \alpha_1^T(k_{m_x}, k_{m_y}, k_z) &= A^T(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) T(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z),
\end{aligned}$$

where

$$A^R = - \frac{16\pi^2 q_g q_s}{k^2} e^{i(q_s z_s + q_g z_g^R)} \tag{47}$$

and

$$A^T = - \frac{16\pi^2 q_g q_s}{k^2} e^{i(q_s z_s - q_g z_g^T)} \tag{48}$$

are expressed as functions of  $k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}$  (the midpoint- and offset-conjugate variables, respectively) and depth wavenumber  $k_z$ . By simply defining

$$\begin{aligned}
\alpha_1^{R'}(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) &\equiv S(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) \alpha_1^R(k_{m_x}, k_{m_y}, k_z) \\
&= A^R R,
\end{aligned} \tag{49}$$

and

$$\begin{aligned}
& \alpha_1^T(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) \\
& \equiv S(k_{m_x}, k_{m_y}, k_{h_x}, k_{h_y}, k_z) \alpha_1^T(k_{m_x}, k_{m_y}, k_z) \\
& = A^T T,
\end{aligned} \tag{50}$$

we generate a new mapping, in which  $\alpha_1^R$  is determined by the reflection data (including its wavelet)  $R$  and the analytic form  $A^R$ . Then,  $\alpha_1^R$  is substituted into equation 50 to determine the transmitted field  $T$  including the wavelet. This scheme, which only depends on  $\alpha_1^R(k_{m_x}, k_{m_y}, k_z) = \alpha_1^T(k_{m_x}, k_{m_y}, k_z)$  (a conjecture and approximation to be fully examined in a future correspondence), does not in any way require knowledge of the wavelet, let alone its phase character. This wavelet independence is in contrast with the methods based on reciprocity theorems, as given in the introduction of this paper.

### Analysis of the linear reflection-to-transmission mapping in one dimension

We examine the reflection-to-transmission linear map procedure described here for the case of a 1D<sup>6</sup> normally incident acoustic plane wave on a delta- or spike-like velocity perturbation in a homogeneous reference medium. Figure 2 illustrates the 1D model and the two experimental configurations we analyze. With source fixed at the origin  $z_s = 0$ , field position at any depth  $z_g$ , and delta-scatterer of amplitude  $\lambda$  placed at  $z_0$ , the full nonlinear expression for the scattered field in terms of these elements is

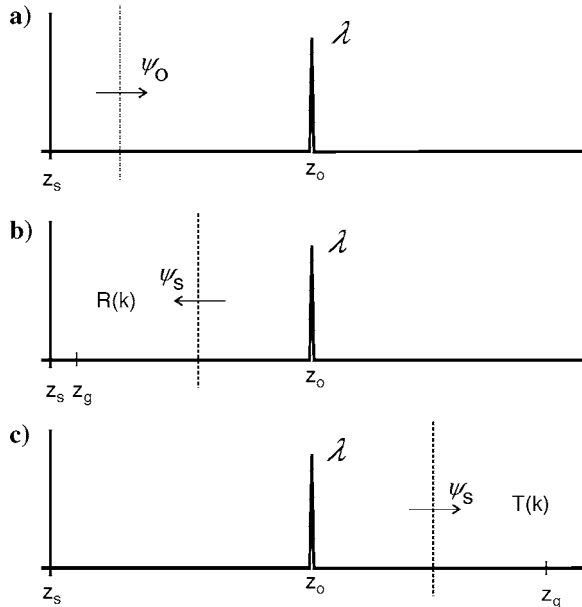


Figure 2. Schematic illustration of the 1D delta-scatterer analytic framework. (a) The incident plane wave  $\psi_0$  propagates toward the scatterer  $\lambda \delta(z - z_0)$ ; (b) the reflection response with  $z_g < z_0$ ; and (c) the transmission response with  $z_g > z_0$ .

<sup>6</sup>This obviously contravenes the requirement for lateral variability that we just mentioned; as a consequence, we shall only be able to analyze and compare the wavenumber  $k = 0$  component of both linear model estimates.

$$\psi_s(z_g, z_s = 0, k) = \frac{\frac{k\lambda}{2i} e^{ik|z_g - z_0|} e^{ikz_0}}{1 - \frac{k\lambda}{2i}}. \tag{51}$$

This general expression is specified to correspond to data of either reflection or transmission type by setting the receiver at depths shallower or deeper than the scatterer (see Figure 2). These cases result in

$$R(z_g, k) \equiv \psi_s(z_g < z_0, z_s = 0, k) = \frac{\frac{k\lambda}{2i} e^{i2kz_0} e^{-ikz_g}}{1 - \frac{k\lambda}{2i}}, \tag{52}$$

$$T(z_g, k) \equiv \psi_s(z_g > z_0, z_s = 0, k) = \frac{\frac{k\lambda}{2i} e^{ikz_g}}{1 - \frac{k\lambda}{2i}}.$$

Given a homogeneous reference medium characterized by wavespeed  $c_0$ , these data are related to their respective linear model components  $\alpha_1^R(z)$  and  $\alpha_1^T(z)$  using

$$R(z_g, k) = \int_{-\infty}^{\infty} G_0(z_g | z'; \omega) k^2 \alpha_1^R(z') \psi_0(z' | 0; k) dz', \tag{53}$$

$$T(z_g, k) = \int_{-\infty}^{\infty} G_0(z_g | z'; \omega) k^2 \alpha_1^T(z') \psi_0(z' | 0; k) dz'.$$

Being sensitive to the relative depths of the source, receiver, and integration variable  $z'$ , the reflection and transmission configurations lead to quite different expressions; with  $G_0(z_g | z'; \omega) = (i2k)^{-1} \exp(ik|z_g - z'|)$  and  $\psi_0(z' | 0; k) = \exp(ikz')$  we have

$$\frac{2i}{k} R(z_g, k) e^{-ikz_g} = \tilde{\alpha}_1^R(-2k), \tag{54}$$

$$\frac{2i}{k} T(z_g, k) e^{-ikz_g} = \tilde{\alpha}_1^T(0),$$

in which  $\tilde{\cdot}$  signifies that quantity,  $\cdot$ , in the conjugate ( $k$ ) domain. The second equation in formula 54 makes it clear why we will, from here on, be forced to restrict ourselves to comments on the  $k = 0$  component of the model. Substituting the analytic data in equations 52 into

these formulas for the reflected and transmitted Born inverse approximations, and considering the reflected case at  $k = 0$ , we have

$$\begin{aligned}\tilde{\alpha}_1^R(0) &= \lambda \\ \tilde{\alpha}_1^T(0) &= \lambda.\end{aligned}\tag{55}$$

In other words, at the only  $k$  value for which comparable results are available for 1D analysis,  $k = 0$ , the component of the model that is linear in the reflected data and the component of the model that is linear in the transmitted data are equal. This is an encouraging result because we want the linear model component of one configuration to reproduce the data of the other configuration. However, no inference about  $k \neq 0$  and/or more complicated models are to be drawn from this initial exercise.

The linear method described in this section is based on several assumptions. At the outset is the assumption that an inverse-scattering series solution is available in terms of either reflected or transmitted data. (By transmitted data, we mean the transmitted portion of the scattered field.) The scattered field, and its transmitted and reflected portions, are all zero when the actual medium corresponds to the reference medium; also, in 2D and 3D the terms of the inverse-scattering series may be constructed for both reflected and transmitted data. Hence, there is reason to believe that it is possible to expand the earth property perturbation in terms of either reflected or transmitted data. We further assume that the linear terms in these two expansions are the same, or at least not too far apart. That is the key assumption in this linear reflection-to-transmission mapping procedure, and its validity and possible usefulness is currently being examined.

If we can establish the validity of this assumption, we may find ourselves with a method of no small applicability because there are very few other assumptions or approximations, linear or otherwise, in the expressions used for  $\alpha_1^R$  and  $\alpha_1^T$  in equations 44<sup>7</sup>. Furthermore, because the reference medium is homogeneous, and is never altered or updated, all of the migrations of the linear inversion procedure require data only on one side of the closed volume where the wavefield is predicted, i.e., either reflected or transmitted data, independent of whether or not the actual medium supports two-way propagation.

In 1D we have benefited from the ability to consider this linear reflection-to-transmission map analytically, but as a result of this dimensional restriction we are restricted to the  $k = 0$  portion of the spectrum of  $\alpha_1^T$ . The last example in this section has demonstrated that the assumption  $\alpha_1^R = \alpha_1^T$  is valid at this portion of the spectrum.

## DISCUSSION

In this section we provide (1) a perspective on the new approaches for the reflection-to-transmission mapping problem proposed in this paper; (2) a discussion on how these new approaches relate to the research program on task-separated subseries of the inverse scattering series and to the migration-inversion philosophy and strategy; and (3) a set of open issues that will be addressed and developed as part of our plan.

In the section on constructing the wavefield at depth, a general formula for  $T(\mathbf{p}, \mathbf{q})$ , for arbitrary  $\mathbf{p}, \mathbf{q}$  and a fixed  $\omega$  is presented as a se-

ries in order of the measured reflection data. Constructing this  $T(\mathbf{p}, \mathbf{q})$  for all  $\mathbf{p}, \mathbf{q}$  and a fixed  $\omega$  is equivalent to computing the scattered wavefield at any source and receiver location in the subsurface. (The source and receiver positions are the conjugates of  $\mathbf{p}, \mathbf{q}$ , respectively; all  $\mathbf{p}, \mathbf{q}$  are 2D or 3D vectors depending on the dimension of variation of the subsurface.) A subset of the formula for  $T(\mathbf{p}, \mathbf{q})$  at any  $\mathbf{p}, \mathbf{q}$ , namely that computed with (1)  $|\mathbf{p}| = |\mathbf{q}| = |\mathbf{k}|$  and (2) the  $z$  components of  $\mathbf{p}$  and  $\mathbf{q}$  (i.e.,  $p_z$  and  $q_z$ ) equal to  $q_s$  and  $q_g$ , respectively, is a series for the transmitted component of the scattered field. In 2D, for instance, where  $k_{g_x} = k_g$  and  $k_{s_x} = k_s$ , the transmitted data are  $T(k_g, q_g, k_s, q_s; \omega) = T_1(k_g, q_g, k_s, q_s; \omega) + T_2(k_g, q_g, k_s, q_s; \omega) + \dots$ ;  $T_n$  is the transmitted data  $n$ th order in the measured reflection data, which is  $T(k_g, -q_g, k_s, q_s; \omega)$ . This is a reflection-to-transmission data map in order of the reflection data. In theory, it requires knowledge of the signature of the source wavelet, but it does not make any assumptions regarding this wavelet and is not limited to wavelets or data with particular (minimum) phase properties. Further examination of these ideas as to their practical implementation is underway.

In the recent work on the inverse-scattering series for imaging and inverting primaries (Weglein et al., 2000, 2001, 2003; Shaw et al., 2001, 2002a, b, 2003a, b, 2004; Innanen and Weglein, 2003; Shaw, 2003, 2005; Innanen, 2005; Liu et al., 2005a, b; Zhang and Weglein, 2005), the location of structure is determined by following the action of the inverse series in determining where each individual mechanical property experiences a rapid variation; the values of the changes in those properties, at those interfaces, are also separately determined.

In the history of seismic migration and seismic inversion, there was a development (e.g., Stolt and Weglein, 1985) that took a different route to the location and parameter estimation problem. Rather than going directly from data to location and magnitude of earth properties, it was suggested that there were conceptual and practical advantages to, first, downward continuing and imaging the wavefield to produce a structural map; second, recognizing that the amplitude of that imaged quantity is related to the local, angle-dependent reflection coefficient; and third, using that coefficient to determine changes in local earth material properties. Among the advantages of this two step approach, labeled migration-inversion or migration before inversion, are (1) the structural map does not require that one follow all the trials and tribulations of each earth property as it finds its correct spatial location; (2) the location of reflectors may only depend on the velocity of wave propagation and not require each separate earth property to be defined; and (3) the petroleum industry has a mature and developed experience in locating structures using wavefield extrapolation and imaging techniques.

Of course, all current migration and migration-inversion algorithms depend upon an adequate velocity model to determine structure. In this paper, the section on predicting the wavefield at depth without the velocity model is providing a first embryonic step toward a response to the migration-inversion strategy for locating structure followed by inversion when the adequate velocity model is unavailable.

We anticipate a strategy and plan as follows: For subsalt plays with often ill-defined or nonexistent images at the target (with all current leading-edge velocity analysis and imaging methods), it is reasonable to start by setting the goal as the determination of a well-defined accurate depth image, or structure map, neglecting earth

<sup>7</sup>These are exact equations relating the data to a specific linear approximate model component, but for our current purposes they are viewed as solutions for curve-fitting factors  $\alpha_1^R$  and  $\alpha_1^T$ .

property and rock and fluid prediction. For that objective, an imaging theory involving multidimensional heterogeneity in acoustic velocity only, i.e., the inverse-scattering imaging subseries for determining the location of rapid changes governed by an acoustic medium (without requiring an adequate velocity model; see previous references) would be a good first step in moving toward a subsalt field data test. Once the subsalt target location issue is successfully addressed, the next level of ambition will arrive, i.e., to use the angle-dependent amplitude of the structure map to determine the local earth mechanical and then rock and fluid properties. At that point, the idea of imaging the actual wavefield in depth without an adequate velocity model, as progressed in this paper, becomes relevant. Equations 35 and 38 are direct calculations of the wavefield at depth that nevertheless could benefit from task-specific insights/algorithms (using, e.g., only the imaging terms) to provide only a structural map from an imaged wavefield at depth. In this strategy we concentrate, as a first pass, on the acoustic location properties of the inverse-scattering series, but anticipate that they will evolve into an imaged wavefield at depth procedure, to be viewed as a new task with its own task-specific subseries. The task-separated stages of inversion will become (1) removal of free surface multiples; (2) removal of internal multiples; (3) production of accurate angle-dependent reflection coefficients (i.e., the scattering operator) at depth; and (4) estimation of earth material properties.

## CONCLUSIONS

The inverse-scattering series is a comprehensive theory and inversion methodology for removing multiples and for imaging and inverting primaries. The theory operates directly in terms of reflection data and an estimate of medium propagation properties, the latter of which is neither assumed to be adequate nor ever changed (e.g., iterated) toward adequacy. Directness means that the theory provides algorithms that output the specific indicated inverse objective without the use of external measures of effectiveness, nor centering itself around the minimization of an objective function. As such, horizontal common image gathers, for instance, or any moveout trajectory or weighted sums thereof — criteria at the heart of many valuable and worthwhile processing methods — are not called upon or required in these direct depth imaging procedures. The inverse-scattering depth-imaging method is the only procedure that states that in principle it is not necessary to know or determine the velocity. In contrast with other methods, no indirect or velocity proxy criteria is sought to satisfy an implicit assumed velocity requirement because the depth imaging method is direct and the velocity is not required. The construction of desired inverse quantities, using task-specific subseries and algorithms derived from the inverse-scattering series, are distinct from methods based on such criteria precisely because of this directness. In this paper, we have described (1) how velocity-independent imaging algorithms can benefit from the availability or prediction of transmission data; (2) how new concepts that originate from the inverse-scattering series can contribute to the satisfaction of that transmission data interest without the typical need for phase assumptions on the reflection data; and (3) how the wavefield at all depths can be predicted without a back-propagation in the actual subsurface.

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## REFERENCES

- Bakulin, A., and R. Calvert, 2004, Virtual source: New method for imaging and 4D below complex overburden: 74th Annual International Meeting, SEG, Expanded Abstracts, 2477–2480.
- Bostock, M., 2004, Green's functions, source signatures, and the normalization of teleseismic wavefields: *Journal of Geophysical Research B, Solid Earth*, **109**, B03303 1–15.
- Claerbout, J. F., 1968, Synthesis of a layered medium from its acoustic transmission response: *Geophysics*, **33**, 204–209.
- de Hoop, M. V., and A. T. de Hoop, 2000, Wavefield reciprocity and optimization in remote sensing: Proceedings of the Royal Society of London, Series A (Mathematical, Physical and Engineering Sciences), **456**, 641–682.
- Duvall, T. L., S. M. Jefferies, J. W. Harvey, and M. A. Pomerantz, 1993, Time-distance helioseismology: *Nature*, **362**, 430–432.
- Fan, C., G. L. Pavlis, A. B. Weglein, and B. G. Nita, 2005, Free surface effect and separation of teleseismic waves: Transmission and reflection, primary vs multiple: AGU/SEG joint meetings, S32A-05.
- , 2006, Removing the free surface multiples of teleseismic transmission and constructed reflection waves using reciprocity and the inverse scattering series: *Geophysics*, **71**, this issue.
- Fokkema, J. T., and P. M. Van den Berg, 1993, *Seismic applications of acoustic reciprocity*: Elsevier Scientific Publication Co.
- Goldberger, M. L., and K. M. Watson, 1964, *Collision theory*: John Wiley & Sons.
- Herman, G. C., 1992, Estimation of the inverse acoustic transmission operator of a heterogeneous medium directly from its reflection operator: *Inverse Problems*, **8**, 559–574.
- Innanen, K. A., 2005, Two non-linear forward and inverse wavefield approximations in the presence of sustained medium perturbations: 75th Annual International Meeting, SEG, Expanded Abstracts, 2362–2365.
- Innanen, K. A., and A. B. Weglein, 2003, Simultaneous imaging and inversion with the inverse scattering series: 8th International Congress of the Sociedade Brasileira de Geofísica, Expanded Abstracts.
- Larose, E., A. Derode, M. Campillo, and M. Fink, 2004, Imaging from one-bit correlations of wideband diffuse wavefields: *Journal of Applied Physics*, **95**, 8393–8399.
- Liu, F., S. A. Shaw, H. Zhang, K. Innanen, Z. Guo, J. Zhang, A. C. Ramírez, D. Foster, K. Matson, R. Keys et al., 2005a, A response to the most pressing seismic E&P challenges: Imaging and inversion beneath a complex, multi-dimensional ill-defined overburden: 75th Annual International Meeting, SEG.
- Liu, F., A. B. Weglein, K. A. Innanen, and B. G. Nita, 2005b, Extension of the nonlinear depth imaging capability of the inverse scattering series to multidimensional media: Strategies and numerical results: Sociedade Brasileira de Geofísica, Expanded Abstracts.
- Malcolm, A. E., J. A. Scales, and B. A. van der Tiggelen, 2004, Retrieving the Green function from diffuse equipartitioned waves: *Physical Review E*, **70**, 015601(R).
- Moses, H. E., 1956, Calculation of scattering potential from reflection coefficients: *Physical Review*, **102**, 559–567.
- Nita, B. G., and A. B. Weglein, 2005, On acoustic reciprocity theorems and the construction of transmission response from reflection data: Mission-Oriented Seismic Research Program, 2004 Annual Report, 271.
- Razavy, M., 1975, Determination of the wave velocity in an inhomogeneous medium from reflection data: *Journal of the Acoustical Society of America*, **58**, 956–963.
- Rickett, J., and J. Claerbout, 1999, Acoustic daylight imaging via spectral factorization: *Helioseismology and reservoir monitoring: The Leading Edge*, **18**, 957–960.
- Roux, P., K. G. Sabra, W. A. Kuperman, and A. Roux, 2005, Ambient noise crosscorrelation in free space: Theoretical approach: *Journal of the Acoustical Society of America*, **117**, 79–84.
- Sabra, K. G., P. Gerstoft, P. Roux, W. A. Kuperman, and M. C. Fehler, 2005, Extracting time-domain Greens function estimates from ambient seismic noise: *Geophysical Research Letters*, **32**, doi: 10.1029/2004GL021862.
- Schuster, G. T., 1999, Seismic interferometric imaging with waveforms, Utah tomography and modeling-migration: Project Midyear Report, 121–130.
- , 2001, Theory of daylight/interferometric imaging, tutorial: 63rd Annual Meeting, EAEG, Extended Abstracts, A032.
- Schuster, G. T., Z. Liu, and F. Followill, 1997, Migration of autocorrelations: 67th Annual International Meeting, SEG, Expanded Abstracts,

- 1893–1896.
- Schuster, G., and J. Rickett, 2000, Daylight imaging in  $V(x, y, z)$  media, Utah tomography and modeling-migration: Project Midyear Report and Stanford Exploration Project Midyear Reports, 55–66.
- Schuster, G. T., J. Yu, J. Sheng, and J. Rickett, 2004, Interferometric/daylight seismic imaging: *Geophysical Journal International*, **157**, 838–852.
- Shapiro, N. M., and M. Campillo, 2004, Emergence of broadband Rayleigh waves from correlations of the ambient seismic noise: *Geophysical Research Letters*, **31**, L07614.
- Shapiro, N. M., M. Campillo, L. Stehly, and M. H. Ritzwoller, 2005, High resolution surface wave tomography from ambient seismic noise: *Science*, **307**, 1615–1617.
- Shaw, S. A., 2001, An inverse-scattering sub-series for predicting the correct special location of reflectors: Initial analysis, testing and evaluation: Presented at the 7th International Congress of the Brazilian Geophysical Society.
- , 2003, An inverse-scattering series and seismic exploration: Presented at the 7th Annual Meeting of the Consortium for the Development of Specialized Seismic Techniques, University of British Columbia.
- , 2005, An inverse scattering series algorithm for depth imaging of reflection data from a layered acoustic medium with an unknown velocity model: Ph.D. thesis, University of Houston.
- Shaw, S. A., and A. B. Weglein, 2003, Imaging seismic reflection data at the correct depth without specifying an accurate velocity model: Initial numerical examples of an inverse scattering subseries in C. H. Chen, ed., *Frontiers of remote sensing information processing*: World Scientific Publishing Co.
- Shaw, S. A., A. B. Weglein, D. J. Foster, K. H. Matson, and R. G. Keys, 2004, Isolation of a leading order depth imaging series and analysis of its convergence properties: *Journal of Seismic Exploration*, **13**, 99–120.
- Shaw, S. A., A. B. Weglein, K. H. Matson, and D. J. Foster, 2001, Understanding coefficients in the 1-D inverse scattering sub-series for imaging — Notes: Mission-Oriented Seismic Research Program, Annual Report, **1**.
- , 2002a, Cooperation of the leading order terms in an inverse-scattering subseries for imaging: 1-D analysis and evaluation: 72nd Annual International Meeting, SEG, Expanded Abstracts, 2277–2280.
- Shaw, S. A., A. B. Weglein, D. J. Foster, K. H. Matson, and R. G. Keys, 2002b, An inverse scattering subseries for predicting the correct location of reflectors: Initial analysis, testing and evaluation: Presented at the 6th Annual Meeting of the Consortium for the Development of Specialized Seismic Techniques, University of British Columbia.
- Shaw, S. A., A. B. Weglein, K. H. Matson, D. J. Foster, and R. G. Keys, 2003a, Convergence properties of a leading order depth imaging series: 73rd Annual International Meeting, SEG, Expanded Abstracts, 937–940.
- , 2003b, Analytical and numerical examples of leading order imaging series: Presented at the 8th International Congress of the Brazilian Geophysical Society.
- Sheng, J., 2001, Migration of multiples and primaries in CDP data by cross-correlogram migration: 71st Annual International Meeting, SEG, Expanded Abstracts, 1297–1300.
- Snieder, R., 2004, Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase: *Physical Review E*, **69**, 046610.
- Snieder, R., A. Gret, H. Douma, and J. Scales, 2002, Coda wave interferometry for estimating nonlinear behavior in seismic velocity: *Science*, **295**, 2253–2255.
- Snieder, R., and S. A. Levin (session organizers), 2004, Oil at the core-mantle boundary? Bridging the gap between exploration and global seismology: AGU Annual Meeting, Session S32A.
- Stolt, R. H., 2002, Seismic data mapping and reconstruction: *Geophysics*, **67**, 890–908.
- Stolt, R. H., and B. Jacobs, 1981, An approach to the inverse seismic problem: SEP Report 25, 121–132.
- Stolt, R. H., and A. B. Weglein, 1985, Migration and inversion of seismic data: *Geophysics*, **50**, 2458–2472.
- Taylor, J. R., 1972, *Scattering theory*: John Wiley & Sons, Inc.
- Wapenaar, K., 2003, Synthesis of an inhomogeneous medium from its acoustic transmission response: *Geophysics*, **68**, 1756–1759.
- , 2004, Retrieving the elastodynamic Greens function of an arbitrary inhomogeneous medium by crosscorrelation: *Physical Review Letters*, **93**, 254301-1–254301-4.
- Wapenaar, K., and J. L. T. Grimbergen, 1996, Reciprocity theorems for one-way wavefields: *Geophysical Journal International*, **127**, 169–177.
- Wapenaar, C. P. A., and F. J. Herrmann, 1993, True-amplitude migration taking fine-layering into account: 73rd Annual International Meeting, Expanded Abstracts, 653–656.
- Wapenaar, K., J. Thorbecke, and D. Draganov, 2004, Relations between reflection and transmission responses of three-dimensional inhomogeneous media: *Geophysical Journal International*, **156**, 179–194.
- Weaver, R. L., and O. I. Lobkis, 2004, Diffuse fields in open systems and the emergence of the Greens function: *Journal of the Acoustical Society of America*, **116**, 2731–2734.
- Weglein, A. B., F. V. Araújo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: *Inverse Problems*, **19**, R27–R83.
- Weglein, A. B., W. E. Boyse, and J. E. Anderson, 1981, Obtaining three-dimensional velocity information directly from reflection seismic data: An inverse scattering formalism: *Geophysics*, **46**, 1116–1120.
- Weglein, A. B., and W. Dragoset, 2005, Multiple attenuation: Reprint Series, Society of Exploration Geophysicists.
- Weglein, A. B., F. A. Gasparotto, P. M. Carvalho, and R. H. Stolt, 1997, An inverse scattering series method for attenuating multiples in seismic reflection data: *Geophysics*, **46**, 1975–1989.
- Weglein, A. B., D. J. Foster, K. H. Matson, S. A. Shaw, P. M. Carvalho, and D. Corrigan, 2001, An inverse-scattering sub-series for predicting the spatial location of reflectors without the precise reference medium and wave velocity: 71st Annual International Meeting, SEG, Expanded Abstracts, 2108–2111.
- Weglein, A. B., K. H. Matson, D. J. Foster, P. M. Carvalho, D. Corrigan, and S. A. Shaw, 2000, Imaging and inversion at depth without a velocity model: Theory, concepts and initial evaluation: 70th Annual International Meeting, SEG, Expanded Abstracts, 1016–1019.
- Weglein, A. B., and R. H. Stolt, 1999, Migration-inversion revisited: *The Leading Edge*, **18**, 950–975.
- Zhang, H., and A. B. Weglein, 2005, The inverse scattering series for tasks associated with primaries: Depth imaging and direct non-linear inversion of 1D variable velocity and density acoustic media: 75th Annual International Meeting, SEG, Expanded Abstracts, 1705–1708.