

Inverse scattering internal multiple attenuation algorithm: an analysis of the pseudo-depth and time monotonicity requirements

Bogdan G. Nita*, Montclair State University and Arthur B. Weglein, University of Houston

SUMMARY

Pseudo-depth monotonicity condition is an important assumption of the inverse scattering internal multiple attenuation algorithm. Analysis reveals that this condition is equivalent to a vertical-time monotonicity condition which is different than the total traveltimes monotonicity suggested in recent literature/discussions. For certain complex media, the monotonicity condition can be too restrictive and, as a result, some multiples will not be predicted by the algorithm. Those cases have to be analyzed in the forward scattering series to determine how the multiples are modeled and to establish if an analogy between the forward and the inverse process would be useful to expand the algorithm to address these kind of events.

INTRODUCTION

The inverse scattering series provides a complete framework for processing primaries and multiples directly in terms of an inadequate velocity model, without updating or in any other way determining the accurate velocity configuration. Algorithms for eliminating free surface and attenuating internal multiples, identified as subseries in the full scattering series, have long been known and applied by the petroleum industry. Here we discuss the multi-dimensional inverse scattering internal multiple attenuation algorithm focusing our attention on the prediction mechanisms. Roughly speaking, the algorithm combines amplitude and phase information of three different arrivals (sub-events) in the data set to exactly predict the time and well approximate the amplitude of interbed multiples.

The inverse scattering internal multiple attenuation algorithm was found through a combination of simple one-dimensional models testing/evaluation and certain similarities between the way the data is constructed by the forward scattering series and the way arrivals in the data are processed by the inverse scattering series. This connection between the forward and the inverse series was analyzed and described in Matson (1996), Matson (1997), Weglein et al. (1997), and Weglein et al. (2003). Specifically, they showed that an internal multiple in the forward scattering series is constructed by summing certain types of scattering interactions which appear starting with the third order in the series. The piece of this term representing the first order approximation to an internal multiple is exactly the one for which the point scatterers satisfy a certain lower-higher-lower relationship in actual depth. Summing over all interactions of this type in the actual medium results in constructing the first order approximation to an internal multiple. By analogy, it was inferred that the first term in the subseries for eliminating the internal multiples would be one constructed from events satisfying the same lower-higher-lower relationship in pseudo-depth. The assumption that the ordering of the actual and the pseudo depths of two sub-events is preserved,

i.e.

$$z_1^{actual} < z_2^{actual} \iff z_1^{pseudo} < z_2^{pseudo}, \quad (1)$$

has been subsequently called “the pseudo-depth monotonicity condition”.

In this paper we further analyze this relation and show that it is equivalent to a vertical or intercept time (here denoted by τ) monotonicity condition

$$z_1^{actual} < z_2^{actual} \iff \tau_1 < \tau_2, \quad (2)$$

for any two sub-events. We also look at the differences between the time monotonicity condition in vertical or intercept time and total travel time. The latter was pointed out by a different algorithm derived from the inverse scattering series in ten Kroode (2002) and further described in Malcolm and de Hoop (2005). We show a 2D example which satisfies the former (and hence is predicted by the original algorithm) but not the latter. Finally we discuss one case in which the monotonicity condition is not satisfied by the sub-events of an internal multiple in either vertical or total travel time and consequently the multiple will not be predicted by either one of the two algorithms. For these cases, the monotonicity condition turns out to be too restrictive and we discuss ways of lowering these restrictions and hence expanding the algorithm to address these types of multiples.

THE INVERSE SCATTERING INTERNAL MULTIPLE ATTENUATION ALGORITHM

The first term in the inverse scattering subseries for internal multiple elimination is (see e.g. Weglein et al. (2003))

$$\begin{aligned} b_3 &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 e^{-iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{iq_2(\varepsilon_g - \varepsilon_s)} \\ &\times \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1(k_g, k_1, z_1) \\ &\times \int_{-\infty}^{z_1} dz_2 e^{i(-q_1 - q_2)z_2} b_1(k_1, k_2, z_2) \\ &\times \int_{z_2}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1(k_2, k_s, z_3) \end{aligned} \quad (3)$$

where $z_1 > z_2$ and $z_2 < z_3$ and b_1 is defined in terms of the original pre-stack data with free surface multiples eliminated, D' , to be

$$D'(k_g, k_s, \omega) = (-2iq_s)^{-1} B(\omega) b_1(k_g, k_s, q_g + q_s) \quad (4)$$

with $B(\omega)$ being the source signature. Here k_s and k_g are horizontal wavenumbers, for source and receiver coordinates x_s

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and x_g , and q_g and q_s are the vertical wavenumbers associated with them. The b_3 on the left hand side represents the first order prediction of the internal multiples. An internal multiple in b_3 is constructed through the following procedure.

The deconvolved data without free-surface multiples in the space-time domain, $D(x_s, x_g, t)$ can be described as a sum of Dirac delta functions

$$D(x_s, x_g, t) = \sum_a R_a \delta(t - t_a) \quad (5)$$

representing different arrivals (primaries and internal multiples). Here R_a represents the amplitude of each arrival and it is a function of source and receiver position x_s and x_g and frequency ω . When transformed to the frequency domain the transformed function $D(x_s, x_g, \omega)$ is a sum

$$\tilde{D}(x_s, x_g, \omega) = \sum_a \tilde{R}_a e^{-i\omega t_a}. \quad (6)$$

Here t_a is the total travelttime for each arrival and it can be thought of as a sum of horizontal and vertical times $t_a = \tau_a + t_{xa}$ (see e.g. Diebold and Stoffa (1981)), where t_{xa} is a function of x_g and x_s . After Fourier transforming over x_s and x_g , the data is $\tilde{D}(k_s, k_g, \omega)$. The transforms act on the amplitude as well as on the phase of the data and transform the part of the phase which is described by the horizontal time t_{xa} . Hence $D(k_s, k_g, \omega)$ can now be thought of as a sum of terms containing $e^{i\omega\tau_a}$ with τ_a being the vertical or intercept time of each arrival

$$\tilde{D}(k_s, k_g, \omega) = \sum_a \tilde{R}'_a e^{-i\omega\tau_a} \quad (7)$$

and where \tilde{R}'_a is the double Fourier transform over x_g and x_s of $\tilde{R}_a e^{-i\omega t_a}$. The multiplication by the obliquity factor, $2iq_s$, changes the amplitude of the plane wave components without affecting the phase; hence $b_1(k_s, k_g, \omega)$ represents an effective plane wave decomposed data and is given by

$$b_1(k_s, k_g, \omega) = \sum_a \tilde{R}''_a e^{-i\omega\tau_a} \quad (8)$$

where $\tilde{R}''_a = 2iq_s \tilde{R}'_a$ and whose phase, $e^{i\omega\tau_a}$, contains information only about the recorded *actual* vertical or intercept time.

Notice that for each planewave component of fixed k_s , k_g and ω we have

$$\omega\tau_a = k_z^{actual} z_a^{actual} \quad (9)$$

where k_z^{actual} is the actual, velocity dependent, vertical wavenumber and z_a^{actual} is the actual depth of the turning point of the planewave. Since the velocity of the actual medium is assumed to be unknown, this relationship is written in terms of the reference velocity as

$$\omega\tau_a = k_z z_a \quad (10)$$

where k_z is the vertical wavenumber of the planewave in the reference medium, $k_z = \sqrt{\frac{\omega}{c_0} - k_s} + \sqrt{\frac{\omega}{c_0} - k_g}$, and z_a is the pseudo-depth of the turning point. This implicit operation in the algorithm is performed by denoting $b_1(k_s, k_g, \omega) = b_1(k_s, k_g, k_z)$ with the latter having the expression

$$b_1(k_s, k_g, k_z) = \sum_a \tilde{R}''_a e^{-ik_z z_a}. \quad (11)$$

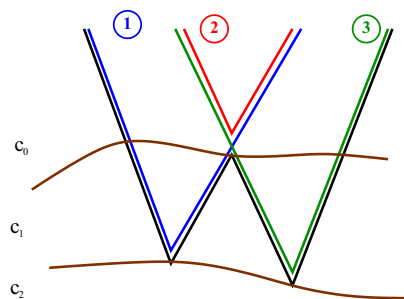


Figure 1: *The sub-events of an internal multiple: the green, blue and red are arrivals in the data which satisfy the lower-higher-lower relationship in pseudo-depths z . The algorithm will construct the phase of the internal multiple shown in black by adding the phases of the green and the blue primaries and subtract the one of the red primary.*

The next step is to Inverse Fourier Transform over the reference k_z hence obtaining

$$b_1(k_s, k_g, z) = \int_{-\infty}^{\infty} e^{ik_z z} b_1(k_s, k_g, k_z) dk_z. \quad (12)$$

Putting together equations (11) and (12) we find

$$b_1(k_s, k_g, z) = \sum_a \int_{-\infty}^{\infty} \tilde{R}''_a e^{ik_z(z-z_a)} dk_z \quad (13)$$

which represents a sum of delta-like events placed at pseudo-depths z_a and hence the b_1 from the last equation is actually $b_1(k_s, k_g, z_a)$. This last step can also be interpreted as a downward continuation on both source and receiver sides, with the reference velocity c_0 , and an imaging with $\tau = 0$, or, in other words, an un-collapsed F-K migration (see e.g. Stolt (1978)). A discussion of differences in imaging with τ and with t was given in Nita and Weglein (2004). Each internal multiple is constructed by considering three effective data sets b_1 and searching, in the horizontal-wavenumber–pseudo-depth domain, for three arrivals which satisfy the lower-higher-lower relationship in their pseudo-depths, i.e. $z_1 > z_2 < z_3$, (see Figure 1 for an example of three such primary events). Having found such three arrivals in the data, the algorithm combines their amplitudes and phases to construct a multiple by adding the phases of the two pseudo-deeper events and subtracting the one of the pseudo-shallower one and by multiplying their amplitudes. One can then see (see e.g. Weglein et al. (2003)) that the time of arrival of an internal multiple is exactly predicted and its amplitude is well approximated by this procedure.

As pointed out in the first section, the lower-higher-lower restriction was inferred from the analogy with the forward scattering series description of internal multiples: the first order approximation to an internal multiple (which occurs in the third term of the series) is built up by summing over all

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scattering interactions which satisfy a lower-higher-lower relationship in actual depth. The assumption that this relationship is preserved in going from actual depth to pseudo-depth is called “the pseudo-depth monotonicity condition”. (Recall that a monotonic function $f(x)$ satisfies $f(x_1) < f(x_2) \iff x_1 < x_2$; here, we regard the pseudo-depth as a function of actual depth). Notice that the lower-higher-lower relationship in pseudo-depth can be translated, from equation (10), in a similar longer-shorter-longer relationship in the vertical or intercept time of the three events. Accordingly, the pseudo-depth monotonicity is also translated in a vertical time monotonicity condition. Notice that this is different from the total time monotonicity assumed by the algorithm introduced in ten Kroode (2002). The latter is employing asymptotic evaluations of certain Fourier integrals which result in an algorithm in the space domain, having a ray theory assumption and the less inclusive total time monotonicity requirement. The justification for this approach was the attempt to attenuate a first order approximation to an internal multiple built by the forward scattering series. In contrast, the original algorithm is aimed at predicting and attenuating the actual multiples in the data and hence it takes into consideration the full wavefield, with no asymptotic compromises, and results in a more inclusive vertical time monotonicity condition.

In the following section we discuss a 2D example in which the geometry of the subsurface leads to the existence of a multiple which satisfies the pseudo-depth/vertical-time but not the total time monotonicity condition.

VERTICAL TIME AND TOTAL TRAVEL TIME MONOTONICITY: A TWO DIMENSIONAL EXAMPLE

Consider the earth model shown in Figure 2. For simplicity we assume that only the density ρ varies at the interface and it has the value ρ_0 in the reference medium and ρ_1 in the actual medium. The velocity is constant c_0 . The actual internal multiple is shown in black and the sub-events composing the multiple are shown in green, blue and red. First, notice that the total traveltime of the shallower reflection (the red event) is bigger than both deeper reflection (green and blue) due to the large offsets needed to record such an event. This implies that the longer-shorter-longer relationship is not satisfied by these particular sub-events in the total traveltime.

Next we calculate the vertical times for individual sub-events. The vertical time for the red event along the left leg is (see Figure 2)

$$\tau_{red}^1 = z_1 \frac{\cos \theta_{in}}{c_0} \quad (14)$$

and along the right leg is

$$\tau_{red}^2 = z_1 \frac{\cos \theta_{out}}{c_0}. \quad (15)$$

Summing the two legs we find the total vertical time along the red event to be

$$\tau_{red} = \frac{z_1}{c_0} (\cos \theta_{in} + \cos \theta_{out}). \quad (16)$$

Similarly, for the green event we have

$$\tau_{green} = \frac{z_2}{c_0} (\cos \phi_{in} + \cos \phi_{out}). \quad (17)$$

Since the velocity is constant, $\theta_{out} = \phi_{out}$; we also have that $\phi_{in} < \theta_{in}$, and hence $\cos \phi_{in} > \cos \theta_{in}$, and $z_2 > z_1$ which results in

$$\tau_{green} > \tau_{red}. \quad (18)$$

It is not difficult to see that similarly, for this example, we have

$$\tau_{blue} > \tau_{red} \quad (19)$$

where τ_{blue} is the vertical time of the blue primary in Figure 2. The conclusion is that for this model and particular internal multiple, the longer-shorter-longer relationship is satisfied by the vertical or intercept times of the three sub-events but not by their total traveltimes. According to equation (10), this relation translates into the lower-higher-lower relationship between the pseudo-depths of the sub-events and hence the internal multiple depicted in Figure 2 will be predicted by the inverse scattering internal multiple attenuation algorithm in Equation (3).

In the next section we discuss an earth model and a particular internal multiple in which the longer-shorter-longer relationship in vertical and total travel time is not satisfied.

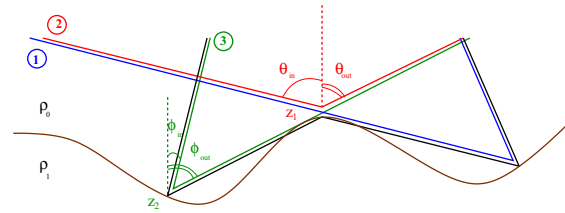


Figure 2: A two dimensional earth model with an internal multiple satisfying the time monotonicity in vertical time but not in total travel time

BREAKING THE TIME MONOTONICITY: A TWO DIMENSIONAL EXAMPLE

Consider the earth model shown in Figure 3 where $c_0 < c_1$ (a similar example was discussed in ten Kroode (2002)). A high velocity zone, in which the propagation speed is c_3 much higher than c_0 , intersects one leg of the internal multiple and hence one leg of one of its sub-events (the blue primary in Figure 3). Due to this high velocity zone and the fact that $c_0 < c_1$, one can easily imagine a situation in which both the total and the vertical time of the blue primary are shorter than the total and vertical times respectively of the red primary (for example when the measurement surface is sufficiently far from the interface). In this case the lower-higher-lower relationship between the pseudo-depths of the sub-events is not satisfied and hence the internal multiple shown in the picture will not be predicted. The monotonicity is in consequence broken, since even though the actual depths still satisfy a lower-higher-lower

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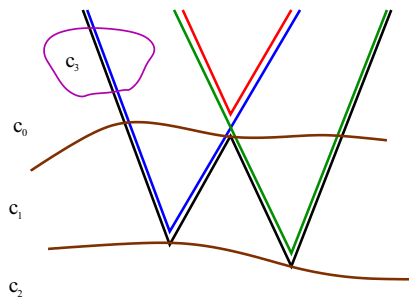


Figure 3: A two dimensional earth model with an internal multiple containing sub-events which do not satisfy the time monotonicity in either total travelt ime or vertical time.

relationship, the pseudo-depths, vertical times or total times of the sub-events do not.

To better understand the multiples which do not satisfy the pseudo-depth / vertical-time monotonicity condition and to expand the algorithm to address them, one has to study their creation in the forward scattering series. As indicated in Matson (1996), Matson (1997) and Weglein et al. (2003) the lower-higher-lower relationship in pseudo-depth z was pointed to by the forward scattering series: the first order approximation to an internal multiple is constructed in the forward scattering series from interactions with point scatterers which satisfy the lower-higher-lower relationship in actual depth. It would be interesting to analyze how a multiple that breaks the monotonicity assumption is constructed by the forward series and to determine if an analogy between the forward and the inverse process would be useful to expand the algorithm to address these kind of events. This particular issue and others will be the subject of future research.

CONCLUSIONS

In this paper we presented an analytic analysis of the inverse scattering internal multiple attenuation algorithm for multi dimensional media. We particularly focused on the mechanism of predicting amplitude and phase properties of interbed multiples. We have discussed in detail the pseudo-depth/vertical-time monotonicity condition and compared it with a similar total travelt ime relation. Furthermore, we showed that this restriction on the sub-events can be too strong and could prevent the prediction of some complex internal multiples.

This research is an important step forward in better understanding the inverse scattering series and the internal multiple attenuation algorithm derived from it. The analytic analysis presented, targets internal multiples which occur in complex multi-dimensional media. Having a better understanding of the structure and definition of such internal multiples opens up new possibilities of identifying, predicting and subtracting them from the collected data. The inverse scattering series is presently the only tool that can achieve these objectives with-

out any knowledge about the actual medium.

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EDITED REFERENCES

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