RESEARCH INTERESTS

Michael A. Jones

My diverse background has led to unconventional techniques, applying functional analysis, geometry, combinatorics, and dynamical systems, to answer questions in mathematics and mathematics applied to the social sciences. I have analyzed cooperative outcomes in repeated Prisoners' Dilemma games, determining that the set of solutions structurally form a cone. I have shown that the cone satisfies 'nice' monotonicity properties and its nonemptiness can be determined by the spectral radius of an associated linear operator. I have also examined repeated Prisoners' Dilemmas from the perspective of reputations in a stochastic environment. This work has ramifications on the compliance of international treaties. By studying the complexity of possible outcomes in repeated games, I have developed a way to compare finite automaton measures of complexity of artificial grammars to the data from psychology experiments.

I have concentrated on how continuous solution concepts can be applied to discrete algorithms from social choice. The techniques have been successful in analyzing the likelihood of outcomes under power indices and how they relate to apportionment methods. The geometry of the social science application provided the genesis of a new dual problem in combinatorics. Additionally, with colleagues, I have looked at ordinal coalition formation as a mathematical analysis of how groups form in political situations. Further, I have applied game theory and probability techniques to solve problems in fair division and to analyze the Wallet Paradox. The following paragraphs introduce and summarize some of my past work, as well as describe plans for future research.

Indefinitely Repeated Prisoners' Dilemmas

To capture different aspects of competition and cooperation, the topic of repeated games has been divided into finitely and infinitely repeated games. Although both of these types of games provide insights for applications, both assume that the players know with certainty when, if ever, the game ends. I examine indefinitely repeated games which introduces uncertainty about the termination date of a game by adding continuation probabilities. A player's continuation probability describes the likelihood that the repeated game continues given the history up to that point. The end of a game may be dictated by budget constraints (as in business or poker), by nature (as in evolutionary games), or even by whim. Thus, the relationship between players' continuation probabilities, whether dependent or independent, permits a wider range of applications and embeds previous work into a more generalized theory; indeed, finitely and infinitely repeated games become endpoints in the spectrum of indefinitely repeated games.

By assuming complete dependence and common knowledge, a time dependent continuation probability yields the likelihood that all of the players play the next round, where the alternative is that the repeated game ends. Payoffs are weighted by the usual discount parameter, as well as the continuation probability. I analyze cooperation in such indefinitely repeated games by examining Prisoners' Dilemma games. Limiting inspection to publicly correlated strategies effectively linearizes the problem of finding subgame perfect equilibria which are equated with vectors in ℓ^∞ that satisfy a matrix inequality. These equilibria form a *cone of cooperation*. The geometry of the cone quantitatively justifies the intuition that "more" cooperation is often supported if the probability of any round of the game continuing increases or if the discount parameter increases; the cone expands and contracts according to changes in these parameters.

For an indefinitely repeated Prisoners' Dilemma game, the existence of cooperative equilibria becomes easy to check. The spectral radius of the matrix becomes a bifurcation value. If a parameter from the stage game is less than the bifurcation value, then cooperative equilibria exist; however, if the parameter is greater than the bifurcation value, then cooperative equilibria do not exist. By an application of the Spectral Mapping Theorem, I find the spectral radius for operators associated with any duration of punishment. Comparing the cones of cooperation and the spectral radii for different operators provides insight into the effect of punishment mechanisms on equilibrium behavior.

I characterize *quasifinite* continuation probabilities which generate repeated games for which cooperative equilibria *never* exist, regardless of the discount parameter and payoffs of the Prisoners' Dilemma stage game. A continuation probability is quasifinite when the asymptotic geometric average of its terms is zero. In future work, I intend to determine the relationship between the asymptotic geometric average and the merging of probability measures, consider the relationship between publicly correlated and uncorrelated strategies in these settings, and analyze the effect of continuation probabilities on other stage games.

Complexity of Artificial Grammars

For repeated games, the complexity of a strategy has been equated with the costs of implementing the strategy; strategies may not be used if the computing or information costs are too high. The literature has focused on what equilibrium outcomes can be achieved under different levels of complexity where more complex strategies can yield more preferable outcomes. Joint work with Erik Bollt (U.S. Naval Academy) concentrates on measuring the complexity of the *possible outcomes* of repeated game strategies. Our motivation is that strategies with more complex outcomes are more difficult to predict. We measure the complexity by computing the topological entropy of the repeated game strategy, using structure borrowed from symbolic dynamics. We applied the techniques developed not only to further examine repeated game outcomes, but also to measure the complexity of artificial grammars, used in mathematical psychology to study implicit learning. I plan to following up this research by comparing the topological entropy and finite automaton measures of complexity to data from psychology experiments.

Fair Division and Game Theory

Dividing objects fairly between people is a problem that is even referred to in the Bible. There is a recreational mathematics variation on fair division that involves dividing a cake among any number of people. The simple solution for two people has one person cut the cake and the other choose. This Cut and Choose Procedure has the problem that it is not equitable, *i.e.*, according to the players' valuations of the cake, they may not believe they are receiving the same amount of cake. Further, Cut and Choose does not yield an equitable outcome, *i.e.*, an outcome where both receive the same amount. I have discovered a symmetric approach to yield an efficient, equitable, and envy-free outcome to cake cutting. This work may be applied to such diverse applications as border disputes and inheritances. Further, the continuous approach has been used to allocate discrete, divisible goods and provides an alternate and simpler allocation that yields the same allocation as the Adjusted Winner procedure. The idea for the cake-cutting article was based on the geometry of the game theoretic analysis of the Wallet Paradox. M. Carroll (University of Scranton), E. Rykken (Muhlenberg College), and I wrote an article to resolve questions raised in an article in the *American Mathematical Monthly*. Our paper proves that there is no optimal strategy for the Wallet Game. This counterintuitive result hinges on the strategy space not being compact.

Power Indices and Combinatorics

By viewing power indices as discrete maps, M. Haines (Augsburg College) and I use the geometry of these maps to determine the likelihood of different outcomes and to relate power indices to apportionment methods. We show that no apportionment method preserves the power, as represented by power indices, of players in a simple weighted-voting game. Since simple weighted-voting games can be used to model political and social interactions (*e.g.*, when the countries of the European Union vote, *etc.*), our work has implications about the optimal design of institutions. Our power index approach to discrete examples motivated a geometric approach to a standard problem in combinatorics. We determined a dual problem associated with the number of nonnegative integer solutions to an equality with inequality constraints. J. Jenq (Montclair) is assisting us in demonstrating the geometry through Java applets for submission to the *Journal of Online Mathematics and Applications*.

Coalition Formation

Further, S. Brams (New York University), D. M. Kilgour (Wilfred Laurier University), and I have developed an ordinal approach to coalition formation. Our work describes a process, as opposed to just an outcome. Our research applies combinatorics to political science and gives a mathematical explanation to the old saw that “Politics make strange bedfellows.” We show that disconnected connected coalitions may form if the preferences of the players are single-peaked, but not if the preferences are more restrictive and are spatially single-peaked. We relate these results to the unfolding of technique of Combs for determining whether stimuli and other psychological data can be represented by unidimensional or multidimensional scales. We equate single-peaked preferences to the qualitative “J” (joint) scales in Combs’ *The Theory of Data* and equate spatially single-peaked preferences to the quantitative “J” scales. We have also used this approach to examine the stability of coalitions. In fact, the likelihood of stable coalitions of a certain size forming under our model has the same shape as coalitions of specific size forming in the US House of Representatives, the Supreme Court, and German parliament.

Reputations, Linkage, and Renegotation Proofness in Repeated Games

G. Downs (New York University) and I are examining how reputation affects compliance in international treaties. By modeling treaties as repeated Prisoners’ Dilemmas with incomplete information, we determine the effect of reputation on cooperation in these games. Our use of stochastic payoffs also gives a salient explanation of renegotiation proofness since our model predicts efficient equilibrium behavior with less than perfect compliance in treaties and not too severe penalties for noncompliance. The geometry of the solution set yields results about the effect of changes in different parameters and how behavior in a treaty can be used to determine outcomes in another treaty. We also consider when linking treaties can increase cooperation.

Diabetic Wound Healing

What may seem like a departure to my regular application of mathematics to the social sciences, I have recently applied mathematics to modeling diabetic wound healing. Motivated by discussions with my best friend who is a diabetic and has had considerable problems with his feet, I realized that the debriding process is similar to harvesting a crop, a problem that I knew a colleague (D. Thomas, Montclair) had modeled successfully. Although we have all experienced wounds, we expect the wound to heal in a reasonable amount of time. Certain segments of the population, however, encounter slow or non-healing wounds due to extenuating factors such as diabetes or poor circulation. When a wound is slow to heal, debris called slough, composed mainly of dead tissue, collects on the wound thereby inhibiting healing. Doctors remove the slough periodically to enhance the healing process. This removal is called debriding the wound. D. Thomas and I designed a differential equation model describing wound healing and incorporated debriding, or the cutting away of slough periodically, in our model by resetting the value of slough to zero. This created a dynamic equation on a time scale. We used debridement as a way to control wound healing and determine a treatment pattern.