

Attached File Supplement to the Findings

Annual Report July 2004:
A Sophomore – Level Transition Course

NSF-DUE-0310753

Internal and External Evaluations of the Course

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Internal Evaluation

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Interim Evaluation Visit Report

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Internal Evaluation:
A Sophomore – Level Transition Course

an NSF-funded Project at
Montclair State University

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June 2004: Internal Evaluation of A Sophomore-Level Transition Course

1 Pre-class survey

Inspired by the work of Palagallo and Blue¹, we designed a nine-item survey to gauge various aspects of students' confidence in mathematics (see Appendix A for the text of the survey). Each item was scored on a five-response Likert scale, with Strongly Agree being 2 points, Agree scoring 1, Neutral scoring 0, Disagree scoring -1, and Strongly Disagree scoring -2. The survey was administered in the 2nd week of class through our course management system (Blackboard 6.0), which ensured anonymity except for letting us know who had and had not finished the survey. The students responded enthusiastically, and without prodding we got a 100% response rate in our population of 18. The pre-class survey served to set a baseline for the students so we can gauge progress. We followed up with a post-class survey in the week before the final exam (see Appendix B for the text of the survey). The post-class survey duplicated the nine items here and then probed for the role of the *Transitions* course in any changes the students experienced.

The averages for the nine responses ranged from 0.22 to 1.00, indicating the students had mild to moderate confidence in their mathematical abilities. The nine items fell in three categories according to scores. Items 2 and 6 were the areas of greatest confidence, with an average score of 1.00 each, indicating that students felt confidence about their ability to read mathematics and felt good about future mathematics courses. In those two areas, no students expressed negative confidence. Items 1, 7, 8, and 9 were the areas of moderate confidence, all averaging scores of 0.78 or 0.56. Thus, their confidence in doing well in the Transitions course lagged slightly behind their confidence in their confidence in future mathematics courses (item 1 vs. item 2). Similarly, the students' confidence in writing and orally communicating mathematics lagged slightly behind their confidence in reading it (items 7 and 8 vs. item 6). Their confidence in using technology to do mathematics lagged behind further (item 9, with a score of 0.56 instead of the 0.78 for the other items). In these four areas, no more than one student expressed negative confidence. However, in the last category, that of slight confidence, five or six students expressed negative confidence. Items 3, 4, and 5 had scores averaging between 0.22 and 0.33. Students were only slightly confident in their ability to apply mathematics to scientific and industrial problems (item 5). They were also only slightly interested in majoring in applied mathematics and having a career involving a lot of applied mathematics (items 3 and 4). See Table 1 for a summary of these results.

Score (SD)	Item #	Item text
<hr/>		
<i>Slight Confidence (5 or 6 negative scores, lowest averages)</i>		
0.22 (1.48)	3	I am interested in majoring in applied mathematics.
0.28 (1.07)	5	I am confident in my ability to apply mathematics to scientific and industrial problems.
0.33 (1.24)	4	I am interested in a career that involves a lot of applied mathematics.
<hr/>		
<i>Moderate Confidence (0 or 1 negative scores, moderate averages)</i>		

¹ J.A. Palagallo, and W.A. Blue, *Analyzing the value of a transitional mathematics course*, In B. Gold, S. Z. Keith and W. A. Marion (Eds.), MAA Notes 49. Assessment practices in undergraduate mathematics, Washington, DC: The Mathematical Association of America, 1999.

0.56 (0.86)	9	I am confident in my ability to use technology to solve mathematical problems.
0.78 (1.00)	1	I am confident about doing well in Math 190.
0.78 (0.73)	7	I am confident in my ability to write mathematics.
0.78 (0.55)	8	I am confident in my ability to orally communicate mathematics.
<i>Greatest Confidence</i> (no negative scores, highest averages)		
1.00 (0.69)	2	I am confident about doing well in my future mathematics courses.
1.00 (0.49)	6	I am confident in my ability to read mathematics.

Table 1: Summary of results for pre-class survey administered during week 2.

One of our purposes for implementing this course at Montclair is to inspire students to major in applied mathematics and consider careers in that area, but these were two of the lowest scoring areas on the survey. We think this is because they have had a dearth of opportunities to see what can be done in the world with applied mathematics, and we are confident our course will engage and interest our students, so we were optimistic about that the post-class survey would show a change of heart.

2 Post-class survey

Fifteen of the original eighteen students participated in the post-class survey; one student had dropped the course, so the response rate was 88%. Eight items on the post-class survey were identical to items on the pre-class survey (Items 2 through 9), which measured students' confidence about and interest in various issues relating to applied mathematics, and one item gauged confidence on having earned a good grade (Item 1). The trend was generally upward compared to the pre-class survey, with results in the mild-to-moderate confidence range, but no changes were statistically significant using a one-tailed t-test at the .1 level. Our optimism about showing a significant change of heart by the average student was unfounded, but individual responses in the free-text portions of the survey did show some individual transformations (see below). The statistical results of the first nine items of the post-test are summarized in Table 2. A comparison to Table 1, the summary of the nine matching pre-class survey items, shows that three items each moved one category, two in the direction of greater confidence: Item 5 moved up to moderate confidence, and Items 1 and 6 switched positions in the greatest and moderate confidence categories.

Score (SD)	Item #	Item text
<i>Slight Confidence</i> (5 negative scores, lowest averages)		
0.33 (1.23)	3	I am interested in majoring in applied mathematics.
0.33 (1.18)	4	I am interested in a career that involves a lot of applied mathematics.
<i>Moderate Confidence</i> (0 or 1 negative scores, moderate averages)		
0.53 (0.74)	5	I am confident in my ability to apply mathematics to scientific and industrial problems.
0.73 (0.59)	8	I am confident in my ability to orally communicate mathematics.
0.80 (0.56)	9	I am confident in my ability to use technology to solve mathematical problems.
0.87 (0.64)	6	I am confident in my ability to read mathematics.

0.87 (0.64)	7	I am confident in my ability to write mathematics.
<i>Greatest Confidence</i> (no negative scores, highest averages)		
1.20 (0.56)	2	I am confident about doing well in my future mathematics courses.
1.27 (0.70)	1	I am confident about doing well in <i>Transitions</i> .

Table 2: Summary of results for post-class survey items identical to pre-class survey administered during week 15.

Eight more items on the post-class survey (Items 2' through 9') probed how much the *Transitions* course increased confidence and interest on those same eight issues relating to applied mathematics covered in Items 2 through 9. On all issues, the average attitude was weak to moderate agreement that the *Transitions* course had a positive effect. These results are summarized in Table 3. There was moderate agreement about increasing confidence in future mathematics courses and reading, writing, and orally communicating mathematics (Items 2', 6', 7', and 8'). There was weakly moderate agreement the course increased understanding how to apply mathematics to science and industry and use technology to solve mathematical problems (Items 5' and 9'). There was weak agreement that the course increased interest in majoring in or seeking a career involving applied mathematics (Items 3' and 4').

Score (SD)	Item #	Item text: <i>Transitions</i> increased my...
<i>Weak</i> (3 or 4 negative scores, lowest averages)		
0.13 (0.92)	3'	...interest in majoring in applied mathematics.
0.27 (0.88)	4'	...interest in a career that involves a lot of applied mathematics.
<i>Weakly Moderate</i> (0 or 1 negative scores, moderate averages)		
0.60 (0.63)	9'	...understanding of how technology is used to solve mathematical problems.
0.67 (0.62)	5'	...understanding of how to apply mathematics to scientific and industrial problems.
<i>Moderate</i> (0 or 1 negative scores, highest averages)		
0.80 (0.56)	8'	...ability to orally communicate mathematics.
0.93 (0.59)	6'	...understanding of how to read mathematics.
0.93 (0.70)	7'	...ability to write mathematics.
1.00 (0.85)	2'	...confidence I will do well in my future math courses.

Table 3: Summary of results for post-class survey items gauging the effect of the *Transitions* course administered during week 15.

The post-class survey included free-response items attached to each of the probes about the effect of the *Transitions* course. The results of those responses are summarized below.

2.1 Weak Agreement, Item 3': *Transitions* increased my interest in majoring in applied mathematics

4 students were already strongly interested in applied math and the course confirmed or slightly enhanced their interest.

4 students were planning other majors and didn't change their minds.

2 liked learning new mathematical techniques and processes involved in applied mathematics for the sake of their mathematical knowledge, but were not interested in becoming applied mathematics majors.

1 didn't like the projects and therefore decided to avoid majoring in applied mathematics.

2.2 Weak Agreement, Item 4': *Transitions* increased my interest in a career that involves a lot of applied mathematics

5 had already decided on other math-related careers, but 1 of these 5 resolved to "find out more information on other careers in mathematics before I make my final decision"

3 already had career plans in applied mathematics and expressed enthusiasm about how helpful they thought this course would be for their future work: "It made me feel more capable of handling problems that I will see in the future."

2.3 Weakly Moderate Agreement, Item 9': *Transitions* increased my understanding of how technology is used to solve mathematical problems

2 mentioned being very happy about learning more technology than just the calculator

2 liked learning more about Excel

4 liked learning Maple: "I think it is a very interesting and useful program and hope I can use it again in future classes"

1 disliked Maple intensely

2 were generally happy about learning technology to reduce unnecessary hand computation or busy-work.

2.4 Weakly Moderate Agreement, Item 5': *Transitions* increased my understanding of how apply mathematics to scientific and industrial problems

2 students mentioned learning how to develop mathematical models.

3 students said their confidence was improved.

2 students said their confidence was improved on some kinds of problems.

2 students admitted they were uncomfortable with and puzzled by modeling.

2.5 Moderate Agreement, Item 8': *Transitions* increased my ability to orally communicate mathematics

1 student's hatred of public speaking overwhelmed any positive feelings

8 students commented positively on their growth in presentation abilities. For 2 of these 8, the course was a transformative experience:

"I just thought math was about using formulas to solve problems and to write in on paper. Through this class I learned how to present mathematics so that the audience understands."

"I think that *Transitions* made me concentrate on communicating what I know and what I have done in order to come to the conclusion I have. Therefore, I am confident that I can do it, where as I was previously not concerned with doing so and was not able to."

2.6 Moderate Agreement, Item 6': *Transitions* increased my understanding of how to read mathematics

8 students noted their ability to read increased, several noting that this was the first class that really made them focus on what was being said: "Most of the problems that we did, didn't

just give you an equation and you had to solve it. You had to read a problem and come up with your own equations.” Others noted that their concentration, speed, or conceptual understanding increased. Even students who had some confidence in their reading ability before taking the class noted that it improved them: “I think that any practice in reading mathematics will increase ability and confidence. Considering *Transitions* did offer practice, it has increased my confidence.”

2.7 Moderate Agreement, Item 7’: *Transitions* increased my ability to write mathematics

6 students noted their writing ability improved, and 2 felt it was a transformative experience: “The ability to write mathematics never even crossed my mind until this semester. I learned how to write math in a very logical and appropriate way.”

“I was not previously exposed to writing mathematics. In *Transitions* I learned how to write a project summary and report. It also made me focus on integrating words into my homework rather than just [doing] the math process.”

2.8 Moderate Agreement, Item 2’: *Transitions* increased my confidence I will do well in my future math courses

2 students were unsure of the value of the course

1 was already confident of future success but appreciated seeing material that would appear in later courses.

4 students’ confidence increased because the course taught them better mathematical reading and writing skills.

2 students’ confidence increased because they learned different ways to think about and solve problems, instead of their focus on “computing an answer” in previous courses.

1 student listed many reasons for his happiness and increased confidence: “It was taught in a way that made me feel secure. The material covered...incorporated a variety of things [from] other math courses. The professors taught the course at a pace that was comfortable and were always well prepared to help and explain the material in depth. This created the confidence and security to ask questions and created excitement to want to learn more.”

2.9 Other Items

Score (SD)	Item #	Item text
1.40 (0.63)	1	<i>Transitions</i> was a valuable course
-0.27 (0.88)	10	Before taking <i>Transitions</i> , I understood the blending of discrete and continuous perspectives
0.80 (0.56)	11	<i>Transitions</i> increased my understanding of the blending of discrete and continuous perspectives
0.73 (0.46)	12	Doing projects in <i>Transitions</i> helped me synthesize and integrate my mathematical knowledge
0.73 (1.03)	13	Doing projects in <i>Transitions</i> increased my ability to work productively on teams

Table 4: Summary of results of post-class survey items not matched with pre-class survey items administered during week 15.

The post-class survey included five other items to probe the students' impressions of the value of the Transitions course, their understanding of discrete vs. continuous models, and their impressions of the projects done. The results and full text of these items are in Table 4.

2.9.1 A valuable course

The highest-scoring item on the post-class survey was Item 1, which measured the value of the course to the students. Comments were universally enthusiastic:

It was a place to grow and become secure of your ability in mathematics.
This course has made me think of math in a different light.
Notation has become easier to decipher and concepts easier to understand.
It has enhanced my ability to read and write mathematics.
I feel like I can write math better.
I learned to communicate math to people who do not understand it as well.
It gave me more insight into making mathematical models (and writing about them).
It is not dealing only with a computation but with real world problems.
The work we did was related to real life situations, which made it more interesting.
I really liked how we took math and applied it to everyday situations.
I acquired skills...that will help me not only in future courses but also in my career.
It has exposed me to other aspects of math that I feel are important to be aware of.
It introduced several topics that will be valuable to me for my future courses.
It helped me understand more about linear and nonlinear systems of equations.
I easily understood the linear algebra chapter about eigenvalues and eigenvectors.
Taking this course along with linear algebra helped me get through linear algebra.
This is a good course [to take] in between calculus and differential equations.
This class covers many topics that the higher-level classes have covered.

For two students, it had a particularly transformative effect:

"When I first decided to take math I thought it was fun. Somewhere along the line I lost that feeling. Well, my feeling about math being fun is now restored!"

"The most valuable thing is the way it helped me see that teaching is not the only option for a math major."

2.9.2 Discrete and Continuous Perspectives

Several students had a small amount of exposure to these perspectives before but the others had no experience with them. After taking the course, three-quarters of the students felt the Transitions course had improved their understanding, even students who had some exposure before: "we went into more depth than I had before" and "Now I know how discrete and continuous perspectives blend."

2.9.3 Projects

The students expressed moderate agreement that the course projects helped them synthesize and integrate their mathematical knowledge, one explicitly noting that they had to use knowledge from other classes to solve the projects. The students expressed moderate agreement that working on teams for the projects was beneficial, although the variation in opinion was much wider: two-thirds of the students liked it – one even had the best project partner since high

school – and the rest were neutral, but one student had a very bad experience (leading to the only score of “strongly disagree” by any student on any item of the survey).

3 Implications

The lack of statistically significant changes in the students’ beliefs about applied mathematics was disappointing, particularly since the purpose of the course is to entice people into majors and careers in applied mathematics. However, there are two mitigating factors that lead us to be hopeful. First, it turned out that a portion of our students were already juniors and seniors who had already firmly decided on their majors. Our survey was geared towards *undecided* sophomores and would get lower scores on key items from students who had already decided upon a career, whether it was to pursue applied mathematics or to pursue teaching mathematics (Items 3, 4, 3’, and 4’ in particular, which also showed the highest standard deviations, implying a divergence in attitude of the decided and undecided students). In future analyses of our course, we will make sure to record their status and analyze its effect on the undecided students separately from its effect on others. Second, the students’ comments in the free-response portion of the survey about the effect of the *Transitions* course were largely positive and reflected the kinds of changes we were looking for. Some students had transformative experiences with respect to mathematics, opening them to a deeper, richer understanding of what it is about and what can be done with it, and many recounted beneficial learning experiences, especially with respect to reading, writing, communicating, and collaborating on mathematics. These are the kinds of changes that we want to see in our students, and we will strive to improve our implementation of the course to strengthen its effects.

Appendix A: The pre-class survey

1. I am confident about doing well in Transitions
2. I am confident about doing well in my future mathematics courses
3. I am interested in majoring in applied mathematics
4. I am interested in a career that involves a lot of applied mathematics
5. I am confident in my ability to apply mathematics to scientific and industrial problems
6. I am confident in my ability to read mathematics
7. I am confident in my ability to write mathematics
8. I am confident in my ability to orally communicate mathematics
9. I am confident in my ability to use technology to solve mathematical problems.

Appendix B: The post-class survey

Part I

The first part of the post-class survey is identical to the pre-class survey except for a change of tense in Item 1. In particular, item 1 for the post-class survey reads: I am confident I *did* well in Transitions while items 2-9 are identical to those in the pre-class survey.

1. I am confident I did well in *Transitions*.
2. I am confident about doing well in my future mathematics courses.
3. I am interested in majoring in applied mathematics.
4. I am interested in a career that involves a lot of applied mathematics.
5. I am confident in my ability to apply mathematics to scientific and industrial problems.
6. I am confident in my ability to read mathematics.
7. I am confident in my ability to write mathematics.
8. I am confident in my ability to orally communicate mathematics.
9. I am confident in my ability to use technology to solve mathematical problems.

Part II

Item 1 is new, while items 2-9 in this portion of the survey are tied directly to corresponding pretest items and are therefore labeled 2'-9'. Items 10 and 11 simulate a pretest-posttest on a topic that most students learned during the course, and items 12 and 13 address projects. Each item is followed by the cue "Please explain your response to the previous item."

1. *Transitions* was a valuable course.
- 2'. *Transitions* increased my confidence I will do well in my future math courses.
- 3'. *Transitions* increased my interest in majoring in applied mathematics.
- 4'. *Transitions* increased my interest in a career that involves a lot of applied mathematics.
- 5'. *Transitions* increased my understanding of how apply mathematics to scientific and industrial problems.
- 6'. *Transitions* increased my understanding of how to read mathematics.

- 7'. *Transitions* increased my ability to write mathematics.
- 8'. *Transitions* increased my ability to orally communicate mathematics.
- 9'. *Transitions* increased my understanding of how technology is used to solve mathematical problems.
10. Before taking *Transitions*, I understood the blending of discrete and continuous perspectives.
11. *Transitions* increased my understanding of the blending of discrete and continuous perspectives.
12. Doing projects in *Transitions* helped me synthesize and integrate my mathematical knowledge.
13. Doing projects in *Transitions* increased my ability to work productively on teams.

Interim Evaluation Visit Report:
A Sophomore – Level Transition Course

an NSF-funded Project at
Montclair State University

By
Mark Parker
June 10, 2004

Introduction:

In 2003, Montclair State University (MSU) was awarded an NSF Course, Curriculum, and Laboratory Improvement (CCLI) grant under the Adaptation and Implementation track. This grant has a total budget of \$52,217 and a 24-month project period from 1 June 2003 through 31 May 2005. The purpose of this grant, DUE-0310753, is to develop curriculum materials for a 3 semester – credit sophomore-level transition course, Math 190, from lower-level mathematics to higher-level mathematics. This report details the external reviewer’s visit to the MSU campus on 4 – 5 June 2004.

The Co-PIs propose to adapt course materials developed under previous NSF – CCLI grants, specifically Project InterMath (NSF – DUE 955414), and making use of ILAPs (NSF – DUE 9455980) and Mount Holyoke’s Laboratory in Mathematical Experimentation (NSF – DUE 9554646).

The objectives of the project are to:

1. introduce real-world applications to students
2. introduce students to experimentation and conjecture
3. introduce computer technology to the students
4. introduce elementary proof techniques in an applied setting
5. standardize the background of students entering higher-level mathematics

Given the fairly traditional course sequence leading up to Math 190, this will be the first exposure most students have to applied mathematics and mathematical modeling. This early exposure may interest more students in the applied mathematics concentration of the Montclair State University mathematics program, and the PIs anticipate that a long – term benefit of Math 190 will be an increase in the number of students enrolled in the applied mathematics concentration.

Project Review:

The primary goal of this interim review is to provide feedback to the PIs on the first offering of this course. A secondary goal is to provide an audit of program activities and expenditures with respect to the proposed activities and budget contained in the original proposal.

Course Demographics

Math 190 was offered for the first time during the Spring semester 2004 and was co-taught by both Co-PIs. A total of eighteen students were initially enrolled, with one later dropping. Of these eighteen initial students, twelve were taking the course after completing the calculus sequence and six others had already completed some upper division coursework, but had not taken algebra or analysis. The six students with the stronger background were allowed to take the course as an upper division elective and were required to complete an extra project. These six students had either already taken, or were taking concurrently, the linear algebra course previously serving as MSU’s introduction to proofs. Most of the students enrolled had already completed the Department’s computer programming requirement.

The course met for 75 – minutes twice a week during the semester. Class time was mix of traditional lecture, worksheet activities, and computer exercises making use of a portable laptop cart as well as Microsoft Excel and Maple. Students were graded on their classroom participation, weekly homework sets, and written and oral group projects.

Students were given pre – and post– surveys via the Blackboard Course Management System to help in the assessment of project goals. All students participated in the pre – survey, and fifteen of the remaining seventeen students participated in the post – survey. Dr. Gideon Weinstein, the MSU internal reviewer, has performed an analysis of this data.

Observations

During the course of my site visit, I met with one of the Co-PIs, Dr. Michael Jones, and the internal reviewer, Dr. Gideon Weinstein. I also reviewed course materials, including the syllabus (Appendix A), schedule (Appendix B), and worksheets, and samples of student work, including projects.

The Co-PIs have a very solid foundation for their course, and I was very impressed with what they have accomplished thus far. Math 190 provides a foundation in applied mathematics which is steeped in discovery learning and technology explorations. Students are given progressively more mature projects throughout the semester, requiring them to integrate more mathematical concepts as they progress. Students coming out of this course should have developed problem solving skills that will enable them to succeed on more open – ended endeavors. As we have found in our program at Carroll College, the introduction of mathematical applications early in a program helps to motivate student interest in and understanding of the theory behind these mathematical concepts.

The Co-PIs have succeeded well in meeting the specific goals outlined in the grant proposal.

Goal 1. introduce real-world applications to students

The entire course is centered around developing discrete and continuous models that represent observed phenomena. In-class activities, homework, and projects all contribute to student exposure to applied mathematics.

Goal 2. introduce students to experimentation and conjecture

The utilization of computers in the classroom allows students to quickly make conjectures and test their hypotheses. This leads directly to the students being able to develop their own understanding of the nature of analytic solutions to different categories of difference equations.

Goal 3. introduce computer technology to the students

Over the course of the semester, students demonstrated their ability to effectively utilize both Excel and Maple as problem solving tools.

Goal 4. introduce elementary proof techniques in an applied setting

Verifying analytic solutions of difference equations leads directly to formalization via induction. Other proof techniques were discussed in class and on homework assignments. Projects focused less on this goal.

Goal 5. standardize the background of students entering higher-level mathematics

As students transition to upper division courses, they will all have exposure to applied mathematics, computer technology, and mathematical formalism. These students now have experience *communicating* about mathematics – via both written reports and oral presentations – a skill vital to our majors.

Focusing on the “adapt and implement” portion of this grant, the Co-PIs have successfully borrowed from the Mount Holyoke *Laboratory in Mathematical Experimentation* in their approach to introducing mathematical formalism, and they have made the jump from the pure mathematics of the Mount Holyoke course to applied mathematics. Perhaps the best lesson students coming out of Math 190 will have learned is that mathematics is a science as well; in that we make observations about behaviors, develop a model for that behavior, and finally formalize (or prove) our model. Although their actual proof experience will not be as great as that of a student coming out of a 3 – credit proof course, the connections that they make between experimentation, discovery, and proof will be of significant benefit for future research efforts.

Although ILAPs did not appear to be directly used in their published form, the central idea to an ILAP has certainly been integrated into Math 190. The Co-PIs projects walk students through the rudiments of an analysis while leaving that analysis somewhat open – ended. At Carroll College, we too have moved

towards developing our own projects that are similar in nature to ILAPs, but without the formal approval process for publication as ILAPs.

The Co-PIs have adapted the most from the Project InterMath materials. In many ways, this course now mirrors changes that we have made to Math 232 at Carroll College. Math 232, a spring – term freshman year course, serves as our introduction to mathematical modeling. We cover difference equations (first order and systems), differential equations (first order), and basic linear algebra. Having experimented with our version of this course at Carroll College, the ambitious syllabus of Math 190 immediately stood out. Dr. Jones remarked that the students struggled with the ties between the discrete and continuous, which we have found to be the case at Carroll as well. One recommendation to address this concern is to drop some material and focus more time specifically on developing discrete and continuous models in parallel (similar to project 3 in Math 190).

The Co-PIs are to be commended for the effort they have made to disseminate information on the project.

- Dr. Jones maintains a web page at <http://www.csam.montclair.edu/%7Ejonesma/transitions.html> which serves as a repository of materials and papers.
- In January 2004, Dr. Jones previewed course information at the NSF Poster Session at the Joint Mathematics Meetings.
- In June 2004, Dr. Mukherjee presented that paper "A Sophomore – Level Transitions Course: Pedagogy, Projects, and Evaluation".
- The paper "A Proofs Course That Transitions Students to Advanced, Applied Mathematics Courses" has been accepted to appear in an MAA Notes volume on *Innovative Methods Beyond Calculus*.

The internal reviewer, Dr. Weinstein, has prepared and carefully analyzed a survey designed to probe students attitudes about their abilities and the course.

Recommendations

In an attempt to improve upon their successes thus far, the Co-PIs should consider the following suggestions.

- **Reduce the content in the syllabus** Eliminating higher – order difference equations and going directly from first order to systems would save 3 – 4 lessons. Eigenvalues could be introduced in terms of matrix powers rather than in terms of the characteristic equation (see Appendix C for a sample worksheet developed by Dr. Holly Zullo at Carroll College).
- **Dedicated Lessons to Proofs** Rather than spending parts of several classes discussing proof techniques, try to combine this time into full lessons.
- **Explicit Use of ILAPs** Both the Lake Pollution ILAP and the Deer and Cougar ILAP would fit within the scope of this course, and one could perhaps be worked in as either a project or extended class exercise to develop student modeling experience.
- **Early Writing Feedback** At Carroll College, we have had good success introducing our students to writing about mathematics by assigning one problem per week as a "Good Problem¹", which requires a short, but formal, write-up. Examples and guidelines to give students are available from Martin Mohlenkamp's web site at <http://www.math.ohiou.edu/~mjm/goodproblems/goodstudent.html> . By getting continual writing feedback during the semester, we have been able to increase the quality of our student project reports. See Appendix D for a sample of the writing guidelines that we give our students in their first semester of our curriculum at Carroll College.
- **Early Reading Assignment** At Carroll College this fall, we will be having our students read an article from the College Math Journal and answering a series of questions designed to get them

¹ See *Good Problems: teaching mathematical writing* , D. Bundy, E. Gibney, J. McColl, M. Mohlenkamp, K. Sandberg, B. Silverstein, P. Staab, and M. Tearle. University of Colorado APPM [preprint #466](#), August 15, 2001

thinking critically about the material they are reading. A draft of this assignment is contained in Appendix E. Again, the point of this exercise is to give students a good example of a mathematics report, and to have them examine the components of a report that we will later ask them to provide.

- **Future Surveys** In the future, if students will be allowed to take Math 190 for upper division credit, they should either be excused from the pre and post surveys or their responses should be separated out.

Future Review Plans:

During the Fall 2004 semester, I will again visit MSU to further review the grant. During this second visit, I will observe the course, talk to past and current students, and discuss the course and its assessment with the Co-PIs and internal reviewer.

Conclusions:

The Co-PI's have developed a strong course which exposes their students to mathematical modeling, writing, problem solving, and the foundations of mathematical formalism. The grant is meeting its goals and the project is thriving under the direction of the Co-PIs.

Appendix A: Spring 2004 Syllabus:

MATH 190-01 – Topics for Undergraduates– Spring 2004

This course meets for one hour and fifteen minutes (2:30-3:45pm), twice a week (M and Th) in Richardson Hall – Room 267. The following information summarizes the course topics, policies, and philosophy. It also lists pertinent information about the textbook, our office hours, and academic honesty.

TEXTBOOK The text for the course is Mathematical Models with Discrete Dynamical Systems by D.C. Arney, F.R. Giordano, and J.S. Robertson (published by McGraw-Hill Custom Publishing). We will supplement these texts with lecture material that may or may not be in the texts, as with content from other sources.

COURSE OVERVIEW This course is to help you transition from the lower-level math courses to the upper-level math courses. The upper-level courses require more creativity and abstraction, as well as concise, analytical thought and the ability to communicate your ideas. Hence, many of the assignments in this course are to let you explore and develop your mathematical intuition, as well as your mathematical maturity. This course will also expose you to many applications of mathematics. Further, we will also be talking and writing mathematics, much more so than in your previous classes. Your participation and insights are valuable to the development of the course – for you and your classmates. And, since there will be classroom activities, it follows that participation will be part of your grade.

YOUR GRADE BROKEN DOWN

Weekly Homework Sets (drop lowest) **250** Participation **050** Projects (2 @ 200; 1 @300) **700**

Letter grades are awarded by the following minimum percentages (divide the total points by 1000):

A- 90%; B- 80%; C- 70%; D- 60%; F < 60.

In lieu of a final exam, the third project will be due on Thursday, May 6, 2004 from 1:00pm – 3:00pm, the time scheduled for our final exam. Because there will be an oral component to this project, all students must be present. This is the first day of finals – so no excuses about having to be out of the country!

This course is designed to get you to communicate. So, to receive credit on any graded event you will have to include relevant explanations in full sentences in your best prose style.

LATE ASSIGNMENTS Late assignments are a pain in the neck to grade. So, to prevent late assignments getting out of hand, no assignment will be accepted late, unless you have a legitimate reason ... for example, a verifiable illness or a death in the family. However, for such a reason, I will notify the Dean of Students of the absence/circumstances and she will confirm the absence/circumstances.

ACADEMIC HONESTY It is presumed that you will do your own work on the homework and projects. Discussing homework problems with others is encouraged; however, submitting work as your own which is copied or paraphrased from someone else is not permitted. Cheating includes, but is not limited to, illegal collaboration, copying, and allowing other students to copy. Anyone found cheating will not be permitted to withdraw and will receive an F for the course. Your academic dean will be informed and a statement will be placed in your permanent file.

OFFICE HOURS

Jones: Mon 1:30 – 2:30 pm Wed 7:45 – 8:15pm Thurs 1:30 – 2:30; 7:45–8:15pm

Mukherjee: Mon 1:30 – 2:30pm; 4:00 – 5:00 pm Thurs 4:00 – 5:00

Specific questions regarding homework or lecture may be asked during office hours. If you cannot make any of the scheduled office hours (above), you may schedule an appointment. You may drop by our offices and see if you can catch us when we are not busy.

MISCELLANY The last day to withdraw and receive a 100% refund is January 27. The final day to receive a 50% refund is February 23. The final day to withdraw from a course is March 11. Please consult the schedule book for more information. Our office information is given below. If you want to leave a message, but do not want to risk running into us; we have mailboxes on the 2nd floor of Richardson Hall. You can leave messages at this location.

Michael A. Jones, Ph.D., Associate Professor; jonesm@mail.montclair.edu; Richardson 206; (973) 655-5448

Arup Mukherjee, Ph.D., Assistant Professor; mukherjeea@mail.montclair.edu; Richardson 209; (973) 655-7243

Appendix B Spring 2004 Course Schedule:

Lesson	Topic/Section
1	Sections 1.1/Intro 1.2; Excel tutorial; Computer sign out procedure
2	Sections 1.2/1.3; In class demonstration; In class assignment
3	Sections 1.3/2.1/2.2; In class demonstration; In class assignment
4	Sections 2.3; Assign Project 1 (Mortgage); Discuss report and presentation
5	Sections 2.4/2.5
6	Section 2.6; Time to work on Project 1 in class
7	Section 2.7; demonstration
8	Section 2.7; Motivation for 6.1
9	Section 6.1/6.2
10	Project 1 due; Project 1 Presentations
11	Sections 6.3/6.4
12	Section 6.5
13	Section 6.6
14	Section 6.7; Assign Project 2 (Mass Spring)
15	Section 6.7
16	Sections 3.1/3.2
17	Project 2 due; Project 2 Presentations
18	Section 4.3
19	Maple and TI Calculator; Section 5.1
20	Sections 5.1/5.2
21	Section 5.3
22	Assign Project 3 (Part higher order DDS and part Diff Eq - Lizards); Section 5.4
23	Section 5.5
24	First Order Differential Equations; Euler's Method (Scheinerman Section 1.1/1.2)
25	Application; Interplay between discrete and continuous
26	Second Order Differential Equations
27	Applications (Return of Mass Spring)
28	Introduction to Nonlinear Applications
29	(Final Exam Period) Presentations for Final Projects 1:00 to 3:00

Appendix C Eigenvalues Worksheet (Dr. Holly Zullo, Carroll College):

MA 232 – Matrix Powers, Eigenvalues, and Eigenvectors

For the next several questions, we will work with the following matrix and vectors:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

1. a. Calculate \mathbf{Ax}

- b. Calculate $\mathbf{A}^2\mathbf{x}$

- c. Calculate $\mathbf{A}^3\mathbf{x}$

- d. Based on what you've seen so far, guess what $\mathbf{A}^{10}\mathbf{x}$ is, and confirm with your calculator.

- e. Write a general expression for $\mathbf{A}^n\mathbf{x}$.

- f. Rewrite $\mathbf{A}^n\mathbf{x}$ so that it is in the form $(\lambda_1)^n\mathbf{x}$ where λ_1 (that's "lambda", not "wave-length thingy") is a scalar (just a number, not a matrix).

Wow! You have just done something pretty incredible – you have written the product of a matrix and a vector as the product of a scalar and a vector! The scalar and the vector that have this property are special enough to be given their own names. λ_1 is an **eigenvalue** of \mathbf{A} , and \mathbf{x} is the corresponding **eigenvector** of \mathbf{A} .

- g. Repeat parts (a) – (f) with \mathbf{y} in place of \mathbf{x} . This time we'll call the scalar λ_2 , just to keep it distinct from the other one.

Amazing! λ_2 is also an eigenvalue of \mathbf{A} , and \mathbf{y} is the corresponding eigenvector.

Are \mathbf{x} and \mathbf{y} linearly independent?

h. Repeat parts (a) – (d) with \mathbf{z} in place of \mathbf{x} . What is different this time?

\mathbf{z} is **not** an eigenvector of \mathbf{A} , so no pretty pattern emerges as you multiply \mathbf{z} by powers of \mathbf{A} . However, all is not lost. Since \mathbf{x} and \mathbf{y} are linearly independent (hopefully that's what you found) and they both have two components, they span \mathbf{R}^2 . That is, any two-component vector can be written as a linear combination of \mathbf{x} and \mathbf{y} .

i. Write \mathbf{z} as a linear combination of \mathbf{x} and \mathbf{y} . (That is, find numbers c_1 and c_2 such that \mathbf{z} can be written as $c_1\mathbf{x} + c_2\mathbf{y}$.)

j. Use what you have found in part (i) to write $\mathbf{A}^n\mathbf{z}$ as $c_1(\lambda_1)^n\mathbf{x} + c_2(\lambda_2)^n\mathbf{y}$.

“So what?” you ask? Well, consider the form of the analytic solution to a homogeneous system of difference equations. If the system is $\mathbf{A}(n+1) = \mathbf{R}\mathbf{A}(n)$ then the analytic solution is $\mathbf{A}(k) = \mathbf{R}^k\mathbf{A}(0)$, which involves evaluating a power of a matrix times a vector. By finding the eigenvalues and eigenvectors of \mathbf{R} , we can rewrite this multiplication as we rewrote a similar multiplication in (j). You're still asking “so what?”? Getting rather demanding, aren't you? There are two answers to that question. One is that rewriting the multiplication in this form will greatly simplify equilibrium analysis for systems of difference equations, since it is generally easier to determine what will happen to a scalar raised to a large power than it is to determine what will happen to a matrix raised to a large power. The other answer is that it can be difficult to raise very large matrices to large powers, and this rewrite allows us to change that problem into one of raising a scalar to a large power, which is considerably easier. Sold yet? Well, even if you're not, it's time to move on and answer the next question that must be bugging you...

How does one go about finding these eigenvalues and eigenvectors anyway???

I'm so glad you asked! We've already seen from the examples that if λ is an eigenvalue of \mathbf{A} and \mathbf{x} is the corresponding eigenvector, then $\mathbf{A}^n\mathbf{x} = \lambda^n\mathbf{x}$.

Rewrite this equation when $n = 1$.

The eigenvalues of \mathbf{A} are the values of λ that result in non-zero solutions to this system, and the eigenvectors are the corresponding solutions. (Does this look familiar? If not, refer back to problem 4.5.2 on page 164.)

2. Now work with the matrix $\mathbf{A} = \begin{bmatrix} -0.2 & 0.6 \\ 0.6 & 0.7 \end{bmatrix}$.

a. Find the eigenvalues of \mathbf{A} . Find the values of λ so that the system $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ will

have nonzero solutions.

- b. For each value of λ found above, find all of the solutions of $\mathbf{Ax} = \lambda\mathbf{x}$. There will be infinitely many solutions for each case, so go ahead and write them in the usual form.
- c. The actual eigenvector for a specific eigenvalue is *any one* of the infinite solutions, except the zero-vector. We usually just pick something simple – I recommend selecting a value of one for the last component. Then determine the rest of the components based on that decision. So determine an eigenvector for each of the eigenvalues.
- d. What is $\mathbf{A}^{10} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$?
- e. If $\mathbf{z} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, then what is $\mathbf{A}^{10}\mathbf{z}$? (Do this without actually computing \mathbf{A}^{10} . Notice that this time we don't have an eigenvector...how did we handle that in problem #1?)

Appendix D Group Project Guidelines (Carroll College):

MA 131 – Group Project Guidelines and Important Dates

Important Dates:

Friday, November 7 – Draft of written report due
November 20 & 21 (tentative) – Oral Presentations (~10 minutes each)
November 21 (tentative) – Final version of written report due

Guidelines:

You must work in groups of 2 or 3 students. Your group will pick **one** of the projects to do. You should turn in one written report for your group, and your oral presentation will be done as a group. Every student should have some part in the oral presentation, and the presentation should be done with Power Point.

Your written report should be done in Microsoft Word, using equation editor, or some other comparable word processor with mathematics capabilities. It must include an **introduction**, a **conclusion**, and **references**. You should use the introduction to briefly summarize the problem in your own words and set the stage for your paper. In the conclusion you should summarize your results and tie things together. Your text book should be listed as a reference, as should any other sources (books or web sites) you consult.

One other requirement for the written report is a **one-page summary**. This should be a brief overview of your entire project, including a very short problem description (shorter than in the introduction), the main methodologies used, and a brief statement of the main results you achieved. Think of this as possibly the only part of your report that a senior manager might read. It needs to clearly and succinctly summarize what you have done.

When you hand in your draft version, it should be as complete as possible in order to receive the maximum benefit from my comments and to receive the maximum points for the rough draft. Any sections that are incomplete should include a note indicating what you still need to include in the section. Ten points will be awarded on the final project for turning in a thorough rough draft. A rough draft that is substantially lacking will not receive full credit. **The commented draft version should be resubmitted with the final version of the report.**

Make sure that you make use of calculus in your projects. For some there may be non-calculus methods that could be used, but part of the point of these projects is to get some practice with calculus. So use it!

These are challenging projects. I encourage each team to come see me for help as needed. It would be a good idea to stop by and show me your calculations once you get started, even if you don't think you need help at that point. That could save you time by keeping you from heading off in a wrong direction.

Some Writing Suggestions

Your project write-up should be done using all of the writing techniques you have been practicing with the Good Problems. This write-up will be longer than any Good Problem, but the type of writing should be similar. A few specific reminders are given below...

Write in the first person, and use the active voice whenever reasonable. For example, rather than, "It was discovered that...", write "We discovered..."

Show your equations and figures clearly. Label all tables and figures, and be sure to explain them in the text. If a figure or table isn't explained in the text, then it should not be included in the paper.

The paper should be more than just a list of answers to homework-style questions – it should tell the story of your project. It is not necessary, nor even desirable, to list the number of the question you are answering. Instead, collect your answers and tell your story with them. Also, feel free to let your personality shine through; it's great to insert humor or invent characters to support your problem scenario.

Spelling and grammar count – proofread carefully!

The introduction and conclusion should *not* be judgments about the problem – refrain from saying, for example, "In conclusion, this was a worthwhile project. I learned a lot." Your introduction should set the stage for your paper – sort of an overview of the problem. Your conclusion should tie together the paper and highlight the major result(s).

Some Presentation Suggestions

Have fun making your slides, but beware of getting so fancy that it detracts from your content. Very plain slides with no moving words are perfectly acceptable.

You will probably not have time during your presentation to discuss every aspect of your problem. Do be sure, though, that you appropriately introduce your problem – remember that the other students aren't necessarily familiar with the project you worked on.

Make sure your presentation is saved on at least two different discs. I have seen several groups in the past have trouble with corrupted discs. You are responsible for having your presentation in working order at the time you are scheduled to present.

Appendix E Reading Questions sample (Dr. Holly Zullo, Carroll College):

Questions about, "Do Dogs Know Calculus?"
(T. Pennings, *College Mathematics Journal*, 34:3, 178-182.)

After reading this article, please answer the following questions.

1. What question is the author trying to answer?
2. What is the author's general approach? (Answer in a few sentences.)
3. What is the title of Figure 1? What information is conveyed by this figure? What is the nature of the discussion of this figure in the text?
4. In equation (1), which letters represent variables and which represent parameters (constants for a specific problem instance)?
5. Following equation (2), the authors says, "if $r < s$, we get no solution." Why is this? Is there any other relationship between r and s that gives no solution?
6. Also after equation (2), the author says, "Finally, note that for fixed r and s , y is proportional to x ." What does " y is proportional to x " mean? If $r = 3$ and $s = 2$, for example, what is the constant of proportionality?
7. What is the title of Table 1? What information is conveyed by this table? What is the nature of the discussion of this table in the text?
8. What assumptions did the author make in solving this problem?
9. What conclusions does the author reach?