A Sophomore–Level Transitions Course: Pedagogy, Projects, and Evaluation[1]

Michael A. Jones, Arup Mukherjee, and Gideon Weinstein
Department of Mathematical Sciences
Montclair State University, Upper Montclair, NJ 07043
Email(s): jonesm@mail.montclair.edu, mukherjeea@mail.montclair.edu, and weinsteing@mail.montclair.edu

Abstract This transitions course aims to help students explore mathematics and conjecture results by using technology to enhance their understanding of mathematical models and to develop students' communication skills through the use of written reports and oral presentation of projects. The course content introduces students to Discrete Dynamical Systems, elementary Linear Algebra and Differential Equations, requiring only Calculus II as a pre-requisite. We include specific projects, course materials, and grading rubrics, as well as attitudinal surveys designed to evaluate the effectiveness of the course.

1 Introduction

The primary purpose of a proofs or transitions course is to introduce concepts and techniques of proof and, thereby, to provide a transition from the lower-level courses to the upper-level courses for mathematics majors. The expected result is that mathematics majors will be more mathematically mature so that they may succeed, appreciate, and benefit from the most challenging upper level courses in the curriculum (*e.g.*, analysis and algebra). At Montclair, the role of a proofs course was played by a 4-credit Linear Algebra course for sophomores and juniors. Because Montclair is a comprehensive university of approximately 10,000 undergraduate students with a large transfer student population, many of our students transfer in credit for Linear Algebra, thereby circumventing our intention of using Linear Algebra as a transitions course. We decided to create a new proofs or transitions course that all students in our program would take. Because we have the flexibility to start from scratch, we wanted to tailor our transitions course to satisfy other needs in our program.

Typically, proof concepts and techniques are introduced in transitions courses through a survey of pure mathematics topics where the explicit content depends on the book(s) used for the course. Many texts focus on number theory, set theory, and axiomatic approaches to examining the real numbers (*e.g.,* Schumacher [9], Eisenberg [3], and Fletcher and Patty [4]). Rotman [7] introduces less new material by concentrating on geometry, trigonometry, and properties of polynomials, but requires a level of precision and rigor that is new to students. Other texts are more ambitious in the content used to introduce proof techniques (*e.g.,* D'Angelo and West [14] use topics from both discrete mathematics–probability, combinatorics, graph theory, sequences, and recurrence relations– and continuous mathematics–series, continuity, differentiation, and Riemann integration). Despite

[1]Partial support for this work was provided by the National Science Foundation's Course, Curriculum, and Laboratory Improvement Program under grant DUE–0310753.

graduating approximately 60 mathematics majors every year, we rarely have more than 3 students apply for Ph.D. programs in a given year.

A large number of students at Montclair want to become mathematics teachers. Many of these students major in pure or applied mathematics, as the competition to get into the teacher education program is high and the requirements for majoring in mathematics education are extensive. Since these students are not enrolled in the teacher education program, they are not fully aware of the demands and requirements of being a mathematics teacher and have little experience in the classroom. As a result, these students do not become mathematics teachers and take positions in industry. We believe that many students who intend on becoming mathematics teachers happen to like mathematics, but are unaware of other career options in mathematics, including graduate school. Because of the make up of our student body, we desire our transitions course not only to serve as an introduction of *how to do* mathematics, but also to introduce students to *what can be done* with mathematics. Our students' exposure to applied mathematics is limited by the restriction that our department does not offer a sophomore-level differential equations course– the only differential equations requirement for our majors is a 4-credit course for juniors and seniors. By informing students at an earlier age of the applicability of mathematics and their career options, we hope to increase the awareness of the different options for majors in our department.

We want our students to gain mathematical maturity in a setting that will expose them to applications of mathematics. Our view of mathematical maturity is less about knowledge of a collection of topics in mathematics, but more about a foundation which the students can build upon and be successful in their upper level courses, whether applied or pure mathematics. As such, content that highlights applications of mathematics can serve as an efficient means to develop mathematical maturity. Indeed, to be successful in the upper level courses, students need to be able to communicate mathematical ideas effectively. This includes both written and oral communication, including presentation and interpersonal communication skills. Our transitions course is designed to develop students' ability to read, write, and speak mathematics. We are not alone in the focus on the ability to communicate mathematics, as there are many transitions texts that highlight proving and writing mathematical results (*e.g.,* Solow [12] and Velleman [13]).

We want our students to learn how pure and applied mathematicians work. As such, our proofs course should help students develop evidence-based conjectures, using different tools to generate examples. Although we want our students to compute examples by hand, we believe that technology provides an efficient means to explore mathematical concepts. Our transitions course should introduce students to the use of technology as a tool to explore mathematics and to generate examples to make conjectures. These conjectures will be the foundation and motivation for their proofs.

We chose difference and elementary differential equations as the content for our transitions course. This provides a natural setting from which to introduce applications of mathematics. Students use Microsoft Excel to generate sequence data from difference equations that they use to make conjectures. The conjectures can be readily proved via direct substitution or induction. The last third of the course parallels the development of difference equations with differential equations, highlighting the relationship between discrete and continuous mathematics. The computer algebra system Maple is also introduced as a way to explore differential equations. The specific course content appears in Appendix A. We require students to turn in weekly homework sets and to pair up to work on three projects that include written and oral components. Details about the pedagogical philosophy of the course appears in Section 2, while a discussion of specific projects appears in Section 3. We discuss the results of a survey given at the beginning of the course in Section 4 and provide copies of the pre-class and post-class surveys in Appendices B and C, respectively.

2 Pedagogy and Philosophy

As motivated in the Introduction, our transitions course is designed to address the specific needs and requirements of students at Montclair State University. The transitions course should enhance the ability of our students to communicate verbally and through writing, contain a heavy technological component and exploratory aspect, teach introductory proof techniques, and introduce students to diverse applications of mathematics. In this section, we discuss the course content and pedagogical underpinnings of the transitions course.

The transitions course combines topics from Difference Equations, elementary Linear Algebra, and Differential Equations, requiring a pre-requisite of Calculus II. Roughly, the first third of the course focuses on first order, linear difference equations (both homogenous and nonhomogeneous equations) and their applications. The second third introduces higher order difference equations and systems of linear difference equations, requiring the introduction of elementary concepts from Linear Algebra. For the first two-thirds of the course, we use *Mathematical Models with Discrete Dynamical Systems* by Arney, Giordano, and Robertson [2]. The last third of the course juxtaposes discrete and continuous formulations by examining first and second order differential equations. A more detailed account of the topics of the course appears in Appendix A.

We motivate the mathematical content by considering relevant applications. A number of the applications from Interdisciplinary Lively Application Projects (ILAPs) by Arney [1] provide the basis for our classroom discussion. ILAPs are integrated, student-centered projects that link mathematics with partner disciplines and originated at the United States Military Academy at West Point through the interaction of the mathematics department with the science, engineering, and social science departments. Not only do the applications demonstrate that mathematics is a tool that can be used to solve and model problems in different fields, but mentioning specific applications provides an opportunity to introduce students to the areas of mathematics in which faculty are currently active. By selecting relevant applications that coincide with the research of the faculty, the students get to know what the faculty are doing and learn about opportunities in undergraduate research. Even though a handful of faculty are already successful in incorporating undergraduates into their research programs, students often become interested in exploring research opportunities too late in his or her undergraduate career.

The ILAPs and other applied problems pique the curiosity of the students and provide a setting in which to examine the results of their mathematical modelling and analysis. As such, the application is the starting point to help them learn the value of exploration, to guide them into forming conjectures from their explorations, and to teach them how to use mathematical reasoning to prove their conjectures. Others have used exploration as a preamble to the mathematical rigors of proof. Indeed, during the 1990's, Mount Holyoke [6] developed a "bridge" course titled *Laboratory in Mathematical Experimentation* which promotes exploration by having students work in small groups, make conjectures, and construct arguments to support the conjectures. Unlike our course, the Mount Holyoke course concentrates on pure mathematics. To develop intuition and make conjectures, we have students use Microsoft Excel to look at the sequence data generated by difference equations under different initial conditions. The numerical or tabular data lets the students conjecture results about long-term behavior before analytically solving for particular solutions to the difference equations. Students are introduced to Maple in the last third of the course to make similar conjectures about the behavior of differential equations. Similarly, students use technology to help them make conjectures about the structure of analytic solutions to difference and differential equations. More specific details about the content of the course and how it relates to proof technique appears in Jones and Mukherjee [5].

As D'Angelo and West [14] mention, a major challenge for students transitioning from lower-level to upper-level courses is the shift of focus from mere computation and calculation to the careful exposition of ideas. To get the students accustomed to writing mathematics, we assign weekly homework sets and discuss the writing of their solutions in class. Although this takes a fair amount of time at the beginning of the term, the students show much and rapid improvement in their writing. Three projects comprise the additional written requirements for the course; the last project serves as a capstone. Students work in small groups (2 or 3 students per group depending on the parity of the number of students in the class). The use of small groups encourages discussion and is a good setting for students to improve their interpersonal communication skills. The projects are more applied than many of the homework problems and consist of different parts and extensions. The projects also have an oral component, as the groups present their work to the other students in the class. Specific projects, details on the project requirements, and a discussion of student work appear in Section 3.

Because we focus on the process of conjecturing results through exploration and on the students' ability to communicate mathematics effectively, we have no tests in the class. The only graded events are weekly homework sets and the three projects. In some sense, the lack of tests alleviates stress and allows the students to spend more time exploring the mathematical concepts. Because the material later in the course builds on the earlier results, we believe that students have to have a working knowledge of the earlier results to do the later assignments.

3 Projects

The three projects dividing the transitions course into approximate thirds (see Appendix A for a detailed course outline) provide a vehicle for students to learn how to conjecture results through exploration, to transition from conjectures to rigorous mathematical results, to sharpen their ability to communicate mathematics, and to develop interpersonal collaboration skills. The projects are designed to be of increasing order of difficulty as the semester progresses, continually challenging the students as they mature. The students work in pairs or groups of three and are responsible for developing mathematical models that appropriately describe the situation detailed in the project, use technology to explore the different possibilities that arise, prepare a detailed written report on their findings, and make a short in-class presentation based on their work. In this section, we provide details of the requirements for technical report writing and give a brief description of the three projects. We also report on student presentations and project reports for project 1.

3.1 Requirements for technical report writing

As mentioned earlier, one of our goals is to help students acquire the ability to write and present mathematics effectively. The students are evaluated on the basis of the mathematical content as well as on their ability to write mathematics well. To this end, we hand out clear instructions for the written report component of each project. A detailed outline of these instructions intermixed with instructor comments about evaluation are included below. The format for our technical reports is a slight perturbation of the format suggested by Kent M. Miller in "Technical Report Format and Writing Guide" in *Interdisciplinary Lively Application Projects* [1]

Title page: Include the title of the project, as well as your names and contact information.

Executive Summary: The purpose of the executive summary is to briefly describe the problem and to summarize the solution(s) and solution method(s). Try to distill the essence of the problem into a document that conveys the information in a clear and concise manner. Think of the executive summary as a stand alone document that is supported by additional materials. A figure of authority in a work place may not have time to pore over every detail and just wants the punch line. The executive summary should be no more than two pages in length. You will be evaluated on the substance, organization, style, and correctness of your executive summary.

A. Substance:

1. Briefly describe the problem and specify the context in which it occurs. Include any simplifying assumptions.
2. Summarize the approach.
3. Summarize the results obtained, both negative and positive as appropriate. Comment on the significance of these results.
4. State briefly whether this problem warrants further examination. If so, recommend the direction of future research should take.

B. Organization: The executive summary should have a logical flow, not just lettered entries! Start by describing the problem in an introductory paragraph. In the case of a single requirement problem, a brief discussion of your findings is also appropriate in the introduction. In the case of a multiple requirement problem, address the solution to each requirement in separate paragraphs.

C. Style: Avoid unnecessary use of the passive voice, slang, technical jargon, and acronyms. The use of simple sentences often communicates the information best. However, avoid your sentences sounding too choppy.

D. Correctness: The document should be free of spelling, grammatical, and punctuation errors.

Appendices: Include one appendix for each major requirement or subsection of the project. Each appendix should be arranged as follows.

A. Problem Statement: The first paragraph should be a concise summary of the problem.

B. Facts Bearing on the Problem: Include statements of undeniable facts having an influence on the problem and/or its solution. Exercise care to exclude unnecessary or extraneous information that may confuse the issue.

C. Assumptions: State any assumptions you made to solve the problem. Assumptions are often needed to simplify the problem and are especially necessary in the absence of actual data. The assumptions, while not facts, must have a basis in fact. Justify your assumptions.

D. Analysis: Present a detailed solution of the problem. Do not just include the mathematics to support your answer. Embed the mathematics and symbols in sentences describing the work and what you are doing. Omit any of the sections below that are not applicable:

1. Definition of variables and symbols.
2. Methodology.
3. Formulas and expressions used and manipulated.
4. Calculations.
5. Diagrams (essential plots and graphs)
6. Discussion of results, referring to numbered equations and numbered figures.
7. References to other supporting documents or annexes.

E. Conclusions: Conclusions must follow logically from the analysis. Do not introduce new material in this section. Answer the problem directly.

Annexes: Include other supporting graphical output, as well as computer programs, data lists, etc. Too extensive data support in the solution of a portion of the project breaks up the flow of your document. For example, including $a(0)$ through $a(10000)$, a tabular or numerical solution to a discrete dynamical system may be of interest, but this should appear as an Annex.

Acknowledgements: Include references to material that was consulted for the project.

3.2 Material covered before project 1

As indicated in the course schedule (see Appendix A), the content for the first third of the semester involves applications that can be modelled using first order discrete dynamical systems. In particular, the students learn how to use Microsoft Excel to investigate the behavior of homogeneous and non-homogenous linear first order discrete dynamical systems with a range of applications from interest rates, savings schemes and retirement planning, to modelling the pollution in a lake and estimating drug dosages. Through their explorations they develop a sense of equilibrium values, stability, and long term behavior of linear first order discrete dynamical systems. The explorations lead to conjectures about the structure of solutions which are verified by direct proof methods. Homework problems vary from routine computations to careful write-up involving simple proof techniques. The first project is assigned during week 3 and the report and presentations are due in week 5. The exact wording for the first project together with comments about student reports and presentations follow.

3.3 Project 1

You work in the back room of JM Real Estate. You are asked to determine a number of different payment schedules for a client of a real estate agent at JM. Unfortunately, the real estate agent has not had the opportunity to run a credit report on the client. This means that you will have to "run numbers" for different interest rates. Also, the interest rate varies according to the length of the mortgage and the potential home owner is considering fixed interest rates on 20-year and 30-year mortgages.

Scenario 1: The client would like to borrow $200,000. Depending on her credit, she can receive a 30-year mortgage at annual interest rates of 5.5 or 6 percent compounded monthly. Determine her monthly payment.

Scenario 2: The client would like to borrow $200,000. Depending on her credit, she can receive a 20-year mortgage at annual interest rates of 5 or 5.35 percent compounded monthly. Determine her monthly payment.

Scenario 3: The client decides that she would like to have a monthly payment of $1050. Determine the maximum amount that she can borrow in a 30-year loan if her annual interest rate is 5.5 or 6 percent compounded monthly.

Scenario 4: The client is indecisive and would like to determine the maximum that she can borrow under a 30-year mortgage at an annual interest rate of 6 percent compounded monthly and a monthly payment of p dollars.

Help the real estate agent by providing a report that determines answers to the above questions. Use your prowess in numerical and analytical solutions for difference equations to determine the answers. Support your work and follow the report format discussed in class.

3.4 Reports and presentations

The reports handed in by the students for the mortgage problem demonstrated that a large percentage of the students were beginning to grasp the concepts behind writing and communicating mathematics well. None of the students had any difficulty with the mathematical content of the problems and many groups were quite innovative in their reports and presentations. In particular, one group came up with fictitious scenarios involving the spouses of the professors looking for mortgages and used this storyline to present their results. In general, the in-class presentations ranged from fair to very good. Student groups who were toward the lower end of the presentation spectrum spent too much time on details and lacked the required planning to stress the important portions carefully. The quality of student reports on the projects was very good. Almost all students based their conclusions on the numerical work using Microsoft Excel, even though their report included correct analytical solutions to the equations. Some groups successfully attempted to generalize their results. In particular, they attempted to use parameters for problems which only varied numerically (for example, in scenario 1 above they attempted to use a interest rate parameter r to develop a formula for the monthly payment and then substituted particular numerical values for r). We feel that the subtle transition between computing the analytic solution correctly to using this effectively to answer meaningful questions is just budding at this stage of the course. This conjecture will only be verified as we carefully monitor the students' progress through the semester.

3.5 Project 2

The ideas introduced for linear first order discrete dynamical systems are extended to second order systems and then to discrete dynamical systems of several variables during the second third of the course. This also includes a basic introduction to linear algebra including matrix multiplication, linear systems, and eigenvalues and eigenvectors. Compartment problems, voting dynamics, mixing problems, and mass spring systems are used as models of real world situations that lead to systems in several variables or second order discrete dynamical systems in one variable.

Project 2 involves developing and analyzing a model for fluctuating gasoline prices. The details of the project are given in Arney, Giordano, and Robertson [2, pp. 441–447]. In particular, the first portion of the project requires the students to develop and analyze a single first order discrete dynamical system that models changes in gasoline prices. The second part requires them to analyze the interplay between supply and prices. This involves using a first order discrete dynamical system involving two variables and the students are required to investigate this system numerically and analytically.

3.6 Project 3

To highlight the difference between modeling behavior discretely and continuously, the third project involves modeling the population dynamics induced by the mating strategies of lizards in California. The cyclic behavior of the populations of lizards with different mating strategies (and distinct colorations) is modeled using evolutionary game theory. For a more detailed explanation of the strategies, placing them into the context of the actual practices of the lizards, consider Sinervo and Lively [11] or the website [10]. When modeled discretely, the evolutionary process can be viewed as a system of difference equations with an attracting fixed point or equilibrium. When modeled continuously, using a system of linear differential equations, the model has a center equilibrium (neither attracting nor repelling). The mathematics of the models are discussed in Jones and Mukherjee [5].

The lizard population data (presented in Sinervo and Lively [11] and on the website [10]) does not show the population converging to the equilibrium, as predicted by the discrete model. Instead, the data exhibits oscillatory behavior, as predicted by the continuous model. By considering the evolutionary model of the lizard population through two different models, students learn that there is no one way to model reality. The discrepancy highlights that models should be developed and compared to real data, if possible. The juxtaposition between the discrete and continuous models naturally leads to a discussion about increasing the complexity of models and how simple models are valuable, too.

4 Evaluation

Inspired by the work of Palagallo and Blue [8], we designed a nine-item survey to gauge various aspects of students' confidence in mathematics (see Appendix B for the text of the survey). Each item was scored on a five-response Likert scale, with Strongly Agree being 2 points, Agree scoring 1, Neutral scoring 0, Disagree scoring -1, and Strongly Disagree scoring -2. The survey was administered in the second week of class through our course management system (Blackboard 6.0), which ensured anonymity except for letting us know who had and had not finished the survey. The students responded enthusiastically, and without prodding we got a 100 % response rate in our population of 18. The pre-class survey set a baseline for the students so we can gauge progress. We will follow up with a post-class survey in the week before the final exam that will measure changes in confidence by duplicating the nine items of the pre-test. However, changes in confidence could have many sources, so the post-class survey includes a second part that probes the role of the transitions course in any changes the students have experienced (see Appendix C for the text of the survey).

The averages for the nine responses ranged from 0.22 to 1.00, indicating the students had mild to moderate confidence in their mathematical abilities. The nine items fell in three categories according to the scores. Items 2 and 6 were the areas of greatest confidence, with an average score of 1.00 each, indicating that students felt confident about their ability to read mathematics and felt good about future mathematics courses. In those two areas, no students expressed negative confidence. Items 1, 7, 8, and 9 were the areas of moderate confidence, all averaging scores of 0.78 or 0.56. Thus, their confidence in doing well in the transitions course lagged slightly behind their confidence in future mathematics courses (item 1 vs. item 2). Similarly, the students' confidence in writing and orally communicating mathematics lagged slightly behind their confidence in reading it (items 7 and 8 vs. item 6). Their confidence in using technology to do mathematics lagged behind further (item 9, with a score of 0.56 instead of the 0.78 for the other items). In these four areas,

Score(SD)	Item #	Item text
		Slight Confidence (5 or 6 negative scores, lowest averages)
0.22 (1.48)	3	I am interested in majoring in applied mathematics
0.28 (1.07)	5	I am confident in my ability to apply mathematics to scientific and industrial problems
0.33 (1.24)	4	I am interested in a carer that involves a lot of applied mathematics
		Moderate Confidence (0 or 1 negative scores, moderate averages)
0.56 (0.86)	9	I am confident in my ability to use technology to solve mathematical problems
0.78 (1.00)	1	I am confident about doing well in Math 190 (transitions)
0.78 (0.73)	7	I am confident in my ability to write mathematics
0.78 (0.55)	8	I am confident in my ability to orally communicate mathematics
		Greatest Confidence (no negative scores, highest averages)
1.00 (0.69)	2	I am confident about doing well in my future mathematics courses
1.00 (0.49)	6	I am confident in my ability to read mathematics

Table 1: Summary of results for pre–class survey administered during week 2.

no more than one student expressed negative confidence. However, in the last category, that of slight confidence, five or six students expressed negative confidence. Items 3, 4, and 5 had scores averaging between 0.22 and 0.33. Student's were only slightly confident in their ability to apply mathematics to scientific and industrial problems (item 5). They were also only slightly interested in majoring in applied mathematics and pursuing a career involving a lot of applied mathematics (items 3 and 4). See Table 1 for a summary of these results.

One of our purposes for implementing this course at Montclair is to inspire students to major in applied mathematics and consider careers in that area, but these were two of the lowest scoring areas on the survey. We think this is because they have had a dearth of opportunities to see what can be done with applied mathematics, and we are confident our course will engage and interest our students, so we remain optimistic about the results of the post-class survey.

5 Conclusion

Proofs or transitions courses do not have to survey pure mathematics or introduce students to logic. Any mathematical content can help students learn to read and write proofs, as long as the class fosters an environment where students can explore, make conjectures, and experience how mathematics is done. This allows the proofs course to become more integrated into the program. For Montclair, this meant that the proofs course should highlight applied mathematics to coincide with faculty interests and the lack of students going on for advanced degrees.

It may be difficult to directly apply our twist on a transitions course, as the conditions present at Montclair dictated the development of the course. Yet, the spirit of the course is easily adaptable to other institutions, as is the focus on applied mathematics. If the unmodified course is to be successful at another institution, then the mathematics program should not offer a sophomore-level differential equations course and the program or department should be centered on applied mathematics. However, switching applied mathematics topics to coincide with faculty research interests should cause no serious alteration of the course. Perhaps the best alternative for schools that offer a sophomore-level differential equations course is to offer such a transitions course instead of the differential equations course. The students would receive some of the content taught in a typical sophomore-level differential equations course, while gaining a broader perspective of mathematics. This alternative will be more attractive for institutions that offer an upper level ordinary differential equations course, but should not be ruled out by institutions that do not because of the potential benefits of this approach.

Institutions that have more of a focus on pure mathematics can still adopt a transitions course that includes the spirit of exploration and selects topics to coincide with the faculty's and program's interests. The exploratory aspect and use of technology can still be employed for a proofs course that surveys advanced pure mathematics. There are even specialized software packages that can be used for abstract algebra (*e.g.*, the GAP software). And, there are discrete math packages for most computer algebra systems.

References

[1] D.C. Arney, *Interdisciplinary Lively Applications Projects (ILAPs)*, Mathematical Association of America, Washington, D.C., 1997.

[2] D.C. Arney, F.R. Giordano, and J.S. Robertson *Mathematical Models with Discrete Dynamical Systems* McGraw–Hill Higher Education, 2001.

[3] M. Eisenberg, *The Mathematical Method: A Transition to Advanced Mathematics*, Prentice Hall, Upper Saddle River, NJ, 1996.

[4] P. Fletcher and C.W. Patty, *Foundations of Higher Mathematics*, 3^{rd} edition, PWS–KENT Publishing Company, Boston, MA, 1995.

[5] M.A. Jones and A. Mukherjee, *A Proofs Course that Transitions Students to Advanced, Applied Mathematics Courses*, In D. Maher (Ed.), MAA Notes: Innovative Methods Beyond Calculus, Washington, DC: The Mathematical Association of America, forthcoming.

[6] Mount Holyoke College, *Laboratories in Mathematical Experimentation*, Springer-Verlag, New York, NY, 1997.

[7] J.J. Rotman, *A Journey into Mathematics: An Introduction to Proofs*, Prentice Hall, Upper Saddle River, NJ, 1997.

[8] J.A. Palagoallo, and W.A. Blue, *Analyzing the value of a transitional mathematics course*, In B. Gold, S. Z. Keith and W. A. Marion (Eds.), MAA Notes # 49. Assessment practices in undergraduate mathematics, Washington, DC: The Mathematical Association of America, 1999.

[9] C. Schumacher, *Chapter Zero: Fundamental Notion of Abstract Mathematics*, 2^{nd} Edition, Addison-Wesley, Boston, MA, 2001.

[10] B. Sinervo, *http://www.biology.ucsc.edu/∼barrylab/* #lizardland, 2003.

[11] B. Sinervo and C.M. Lively, "The rock-scissors-paper game and the evolution of alternative male strategies," *Nature*, 340 (1996) 240-246.

[12] D. Solow, *How to Read and Do Proofs: An Introduction to Mathematical Thought Processes*, 3^{rd} Edition, Wiley & Sons, Inc., New York, NY, 2002.

[13] D.J. Velleman, *How to Prove It: A Structured Approach*, Cambridge University Press, Cambridge, MA, 1994.

[14] D.B. West and J.P. D'Angelo, *Mathematical Thinking: Problem-Solving and Proofs*, 2^{nd} Edition, Prentice Hall, Upper Saddle River, NJ, 2001.

Appendix A: Course Schedule

Week 1–4: Introduction to discrete dynamical systems (dds's) and models. Introduction to Excel and its use to investigate numerical solutions of first order dds's. Analytic solutions, equilibrium values and stability, and long term behavior of homogeneous and non–homogeneous linear first order dds's. Conjecturing solutions through exploration and verifying the conjectures through direct proofs is stressed. Project 1 on mortgage calculations assigned at the beginning of week 3.

Week 5: Reports on project 1 due. In class presentation by students.

Week 5–10: Homogeneous and non-homogeneous linear second order dds's. Numerical investigations using Excel, conjecture and verification of analytic solutions, equilibrium values, and long term behavior. Classification of systems of difference equations. Introduction to matrices and use of graphing calculators and Maple to perform matrix operations. Homogeneous and non-homogeneous linear dynamical systems in several variables including solutions using matrices, conjecture and verification of solutions, equilibrium, stability, and long term behavior. Project 2 on gasoline prices which involves modelling using systems is assigned during week 7.

Week 10 Reports on project 2 due. In class presentation by students.

Week 11-15: Interplay between discrete and continuous evolving into first order differential equations and numerical solutions using Euler's method. Second order differential equations. Revisit applications that can be modelled using discrete and continuous models. Introduction to nonlinear models and applications. Project 3 from evolutionary game theory that depends on the discrete or continuous formulation assigned during week 12.

Week 16 Finals week. Presentations for final projects.

Appendix B: Pre–class survey

1. I am confident about doing well in transitions.
2. I am confident about doing well in my future mathematics courses.
3. I am interested in majoring in applied mathematics.
4. I am interested in a career that involves a lot of applied mathematics.
5. I am confident in my ability to apply mathematics to scientific and industrial problems.
6. I am confident in my ability to read mathematics.
7. I am confident in my ability to write mathematics.
8. I am confident in my ability to orally communicate mathematics.
9. I am confident in my ability to use technology to solve mathematical problems.

Appendix C: Post–class survey

Part I

The first part of the post–class survey is identical to the pre–class survey except for a change of tense in item 1. In particular, item 1 for the post–class survey reads: I am confident I *did* well in Transitions while items 2–9 are identical to those in the pre–class survey.

Part II

Items 2–9 in this portion of the survey are tied directly to the corresponding pretest item, items 10 and 11 simulate a pretest/posttest comparison on topics that students learned during the course, and items 12 and 13 address projects. Each item is followed by the cue "Please explain your response to the previous item."

1. Transitions was a valuable course.
2. Transitions increased my confidence that I will do well in my future math course.
3. Transitions increased my interest in majoring in applied mathematics.
4. Transitions increased my interest in a career that involves a lot of applied mathematics.
5. Transitions increased my understanding of how to apply mathematics to scientific and industrial problems.
6. Transitions increased my understanding of how to read mathematics.
7. Transitions increased my ability to write mathematics.

8. Transitions increased my ability to orally communicate mathematics.
9. Transitions increased my understanding of how technology is used to solve mathematical problems.
10. Before taking Transitions, I understood the blending of discrete and continuous perspectives.
11. Transitions increased my understanding of the blending of discrete and continuous perspectives.
12. Doing projects in Transitions helped my synthesize and integrate my mathematical knowledge.
13. Doing projects in Transitions increased my ability to work productively on teams.