Abstract

This grant (NSF DUE 0310753; CCLI A&I) was awarded to develop, adapt, and implement a sophomore-level transition course integrating topics from Linear Algebra, Discrete Dynamical Systems, and Differential Equations. Our objective is to provide a lower-level course where mathematics majors gain experience using technology (specifically, Microsoft Excel and Maple), explore mathematics and conjecture results, apply mathematics to different settings, and determine and communicate the implications of mathematical models while introducing techniques of proof. The students will write reports about and make oral presentations on projects throughout the term and on a capstone project at the end of the term. The projects and materials for the course are adapted from Interdisciplinary Lively Application Projects (NSF-DUE-9455980) and Project Intermath (NSF-DUE-954414). The transition course also contains exploratory aspects of Mount Holyoke's Laboratory in Mathematical Experimentation (NSF-DUE-9554646).

Fertile Ground for a New Course

- Uneven backgrounds of students in our upper-level courses
- Inexperienced upper-level students have little or no exposure to proofs (except in Linear Algebra)
- Transfer students take Linear Algebra at community colleges
- Students are inexperienced in computer algebra systems and spreadsheet programs
- Students unaware of applications and career options
- No required differential equations course
- Few undergraduate students go onto graduate school

Motivation and Inspiration

- While in a 3-year position at the United States Military Academy at West Point, Jones taught courses that integrated discrete and continuous mathematics in calculus while focusing on applications; this included the use of ILAPs.
- Mukherjee attended a workshop at Mount Holyoke explaining their approach to increasing students' exploration in mathematics.
- Jones and Mukherjee attended a workshop at Carroll College that explained further integrations of topics and uses of technology that were developed as part of Project InterMath.

Timeline

- Summer/Fall 2003: Develop course; Recruit
- Spring 2004: Run first section; Collect/analyze data
- Summer 2004: Prepare for second iteration
- Fall 2004: Run second section; Collect data; *etc.*
- Spring 2005: Run third iteration; Collect/analyze data; Make recommendations to department about requiring the course for mathematics majors
- Compare students who took the new course to those students who did not and to past classes of math majors

Sample Content: Induction

- Prove the general form of a solution to a second-order, linear homogeneous difference equation
- $a(n) = s\,a(n-1) + t\,a(n-2)$ where s and t are constants has characteristic polynomial:

$$x^2 = sx + t$$

If r_1 & r_2 are distinct roots:

- Conjecture: $a(n) = c_1 r_1^{\,n} + c_2 r_2^{\,n}$
- And,

$$\begin{aligned} a(n+1) &= s\,a(n) + t\,a(n) \\ &= s\,(c_1 r_1^{\,n} + c_2 r_2^{\,n}) + t\,(c_1 r_1^{\,n-1} + c_2 r_2^{\,n-1}) \\ &= c_1(s\,r_1^{\,n} + t\,r_1^{\,n-1}) + c_2(s\,r_2^{\,n} + t\,r_2^{\,n-1}) \\ &= c_1 r_1^{\,n+1} + c_2 r_2^{\,n+1} \end{aligned}$$

If $r_1 = r_2 = r$ then $s = -2r$ and $t = -r^2$ and:

- Conjecture:

$$a(n) = c_1 r^n + c_2 r^n n$$

And,

$$\begin{aligned} a(n+1) &= s\,a(n) + t\,a(n) \\ &= s\,(c_1 r^n + c_2 r^n n) + t\,(c_1 r^{n-1} + c_2 r_2^{\,n-1}(n-1)) \\ &= c_1(s\,r^n + t\,r^{n-1}) + c_2(s\,r^n n + t\,r^{n-1}(n-1)) \\ &= c_1(2r^{n+1} - r^{n+1}) + c_2(2r^{n+1} n - r^{n+1}(n-1)) \\ &= c_1 r^{n+1} + c_2 r^{n+1}(n+1) \end{aligned}$$

Sample Content: Discrete vs. Continuous

- Evolutionary Game Theory Model of Mating Strategies of Californian Lizards; modeled using Replicator Dynamics for Rock-Paper-Scissors
- Discrete Model; Let a(n), b(n), and c(n) be the populations of three types of lizards

Evolution given by:

$$\begin{bmatrix} \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} a(n) \\ b(n) \\ c(n) \end{bmatrix} = \begin{bmatrix} a(n+1) \\ b(n+1) \\ c(n+1) \end{bmatrix}$$

Consequence:
Attracting Fixed Point $[1/3\ 1/3\ 1/3]^T$

- Continuous Model

Evolution given by:

$$\begin{aligned} a' &= (a + 2c - 1)a \\ b' &= (b + 2a - 1)b \\ c' &= (c + 2b - 1)c \end{aligned}$$

Consequence:
Center Fixed Point $[1/3\ 1/3\ 1/3]^T$

Tentative Schedule for Spring 2004

Tentative Schedule for the Math 190 - Spring Semester 2004 Text: Mathematical Models with Discrete Dynamical Systems by Arney, Giordano, and Robertson (McGraw Hill Custom Publishing, 2001)		
Day	**Date**	**Topic/Section**
Th	22-Jan	Sections 1.1/Intro 1.2; Excel tutorial; Computer sign out procedure
M	26-Jan	Sections 1.2/1.3; In class demonstration; In class assignment
Th	29-Jan	Sections 1.3/2.1/2.2; In class demonstration; In class assignment
M	2-Feb	Sections 2.3; Assign Project 1 (Mortgage); Discuss report and presentation
Th	5-Feb	Sections 2.4/2.5
M	9-Feb	Section 2.6; Time to work on Project 1 in class
Th	16-Feb	Section 2.7
M	23-Feb	Project 1 due; Project 1 Presentations
Th	26-Feb	Sections 6.1/6.2
M	1-Mar	Sections 6.3/6.4
Th	4-Mar	Sections 6.5/6.6
M	8-Mar	Section 6.7; Assign Project 2 (Mass Spring)
Th	11-Mar	Section 6.7
		SPRING BREAK! No classes March 15 and March 18.
M	22-Mar	Sections 3.1/3.2
Th	25-Mar	Project 2 due; Project 2 Presentations
M	29-Mar	Section 4.3
Th	1-Apr	Maple and TI Calculator; Section 5.1
M	5-Apr	Sections 5.1/5.2
Th	8-Apr	Section 5.3
M	12-Apr	Assign Project 3 (Part higher order DDS and part Diff Eq - Lizards); Section 5.4
Th	15-Apr	Section 5.5
M	19-Apr	First Order Differential Equations; Euler's Method (Scheinerman Section 1.1/1.2)
Th	22-Apr	Application; Interplay between discrete and continuous
M	26-Apr	Second Order Differential Equations
Th	29-Apr	Applications (Return of Mass Spring)
M	3-May	Introduction to Nonlinear Applications
Th	**6-May**	**Presentations for Final Projects 1:00 to 3:00**

Attitudinal Survey: Pretest

To be administered week 2 on Blackboard. Week 15 version has additional items measuring impact of the course. The original questions are all repeated verbatim [except #1 being rephrased], so we have two ways to measure increases from the course – by comparing identical questions and by looking at responses to the new items in part 2 of the posttest. All Part 2 posttest items have blank boxes for text responses and the cue: 'Please explain your response to the previous item.'

SCORING SA=strongly agree, A=agree, N=neutral, D=disagree, SD=strongly disagree

PRETEST, week 2

1. I am confident about doing well in *Transitions*
2. I am confident about doing well in my future math courses
3. I am interested in majoring in applied mathematics
4. I am interested in a career that involves applied math
5. I understand how to apply mathematics to scientific and industrial problems
6. I understand how to read mathematics
7. I understand how to write mathematics
8. I am confident in my ability to orally communicate mathematics
9. I understand how technology is used to solve math problems

Attitudinal Survey: Posttest

POSTTEST Part 1, week 15 (Identical to PRETEST except for wording of #1)

1. I am confident I *did* well in *Transitions*mathematics

POSTEST Part 2 [#2-9 are tied directly to corresponding pretest item, #10 “fakes” a pretest item]

1. *Transitions* was a valuable course
2. *Transitions* increased my confidence I will do well in my future math courses
3. *Transitions* increased my interest in majoring in applied math
4. *Transitions* increased my interest in a career that involves a lot of applied math
5. *Transitions* increased my understanding of how to apply math to scientific and industrial problems
6. *Transitions* increased my understanding of how to read math
7. *Transitions* increased my ability to write mathematics
8. *Transitions* increased my ability to orally communicate math
9. *Transitions* increased my understanding of how technology is used to solve mathematical problems
10. Before taking *Transitions*, I understood the blending of discrete and continuous perspectives
11. *Transitions* increased my understanding of the blending of discrete and continuous perspectives
12. Doing projects in *Transitions* helped me synthesize and integrate my knowledge of math
13. Doing projects in *Transitions* increased my ability to work productively in a team setting