To discuss…

- Color Science
- Color Models in image
Light is an electromagnetic wave
- It’s color is characterized by its wavelength
- Laser consists of single wavelength

Most light sources produce contributions over many wavelengths, contributions that fall in the visible wavelength can be seen

λ vs Spectral power curve is called spectral power distribution $E(\lambda)$

Light from 400 to 700 nanometer ($10^{-9}$ meter)
Color Science – Light & Spectra [2]

The diagram illustrates the spectrum of light, ranging from gamma rays to radio waves. The visible light spectrum is highlighted, extending from 380 nm to 750 nm. The wavelengths increase from left to right, with the corresponding energy levels increasing as well.
Red light has longer wavelength in the visible light
Blue light has the shorter.
The shorter the wavelength, higher the vibration & energy
Red photons carry around 1.8eV & blue 3.1eV
Color Science – Vision & Sensitivity

- Eye resembles a camera focusing image into the retina
- Retina consists of rods & 3 kinds of cones
- Rods have more sensitivity when the light level is low & produce image in gray
- The higher the light level, the more the sensitivity with the cones & more neurons firing
- The 3 kinds of cones are more sensitive to R, G & B present in the ratios 40:20:1
Color Science – Vision & Sensitivity [2]

- Eye is most sensitive to the middle of the visible spectrum
- Let us denote the spectral sensitivity of R, G, B cones as a vector \( q(\lambda) \)

\[
q(\lambda) = [q^R(\lambda), q^G(\lambda), q^B(\lambda)]^T
\]
The sensitivity of each cones can be specified as

\[ R = \int E(\lambda) q^R(\lambda) \, d\lambda \quad (1) \]

\[ G = \int E(\lambda) q^G(\lambda) \, d\lambda \quad (2) \]

\[ B = \int E(\lambda) q^B(\lambda) \, d\lambda \quad (3) \]

- \( \int \) - integral

Equations 1, 2, 3 quantify the signals transmitted to the brain.
Equations 1, 2 & 3 applies for self luminous objects, but we mostly receive image light reflected from objects.

The reflectance $S(\lambda)$ varies from object to object.

The formations would be as follows:

- Light from illuminant with SPD $E(\lambda)$ falls on the object with reflectance $S(\lambda)$ and is reflected, then filtered by the eyes sensitivity functions $q(\lambda)$ & the image formation model is now:

  - $R = \int E(\lambda) S(\lambda) q^R(\lambda) \, d\lambda$  
    \hspace{1cm} \text{--------- (4)}
  
  - $G = \int E(\lambda) S(\lambda) q^G(\lambda) \, d\lambda$  
    \hspace{1cm} \text{--------- (5)}
  
  - $B = \int E(\lambda) S(\lambda) q^B(\lambda) \, d\lambda$  
    \hspace{1cm} \text{--------- (6)}

  - $\int$ - integral
Color Science — Gamma Correction

- The RGB in the image files are converted to analog & drive the electron guns of CRT
- The light emitted from CRT is not linear to the voltage applied, it is proportional to voltage raised to the power, called gamma
- This effect can be precorrected by applying inverse transformation
- SMPTE (Society of Motion Picture and Television Engineers) set this value to 2.2
Color Science – Gamma Correction [2]

**FIGURE 3.7**
(a) Linear-wedge gray-scale image.  
(b) Response of monitor to linear wedge.  
(c) Gamma-corrected wedge.  
(d) Output of monitor.

From digital Image Processing C.Gonzalez & R.Woods
Color Science — CIE Chromaticity Diagram

- In 1931, CIE (Commission Internationale de L’Eclairage) combined all results of color matching experiments in the form of CIE color matching functions.

- The reason why some parts of the color matching functions curve are negative?
  - Shift color primaries to the other side to produce those colors.

Multimedia Computing
(CSIT 410)
The CIE decided to use imaginary lights that had two especially nice features:

- one of the lights is “gray” and provides no hue information;
- the other two lights have zero luminance and provide only hue

The response for these three lights is defined by the triple \((X, Y, Z)\) where \(Y\) is the **luminance** (brightness/perceived intensity)
The formula for the CIE *tristimulus* values \((X, Y, Z)\) is

\[
X = \int E(\lambda) \ x(\lambda) \ d \lambda \quad (7)
\]
\[
Y = \int E(\lambda) \ y(\lambda) \ d \lambda \quad (8)
\]
\[
Z = \int E(\lambda) \ z(\lambda) \ d \lambda \quad (9)
\]
Factoring out luminance to concentrate on color, we get:

- $x = \frac{X}{X+Y+Z}$
- $y = \frac{Y}{X+Y+Z}$
- $z = \frac{Z}{X+Y+Z}$

Now $z = 1-x-y$, means $z$ is redundant.

Plotting $x$ vs. $y$ for colors in the visible spectrum we get the CIE Chromaticity diagram.
The Line Joining red to violet is called *purple line*, and is not a part of the spectrum.

Interior points specify all the visible color combinations.

Standard approximations of day light by CIE includes:
- illuminant C (0.310063, 0.316158)
- D65 (0.312713, 0.329016)
- D100
So can you get X, Y & Z from x, y & z?

- No.
- For complete description of color, we need x, y & Y. So,

\[
(X, Y, Z) = \left( \frac{XY}{y}, Y, \frac{(1-x-y)Y}{y} \right).
\]

- x, y represent chromaticity & Y represent luminance
The CIE Chromaticity diagram is found useful in the following situations:
- Comparing *color gamuts* of different set of primaries
- Identifying *complementary colors*
- Determining *dominant wavelength & purity* of different colors
Color Science – Color Monitor Specifications

- When R, G, B electron guns are fired at the maximum voltage (normalized to [0…1]), white point should appear.
- Color monitor specifications includes the white point chromaticity desired

<table>
<thead>
<tr>
<th>System</th>
<th>Red</th>
<th>Green</th>
<th>Blue</th>
<th>White Point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_r$</td>
<td>$y_r$</td>
<td>$x_g$</td>
<td>$y_g$</td>
</tr>
<tr>
<td>NTSC</td>
<td>0.67</td>
<td>0.33</td>
<td>0.21</td>
<td>0.71</td>
</tr>
<tr>
<td>SMPTE</td>
<td>0.630</td>
<td>0.340</td>
<td>0.310</td>
<td>0.595</td>
</tr>
<tr>
<td>EBU</td>
<td>0.64</td>
<td>0.33</td>
<td>0.29</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Chromaticities & white point of monitor specifications
Gamut refers to the subset of colors which can be accurately represented in a given color space or by a certain output device.

Colors are said to be *out of gamut*, if it is perceivable by the user & is not representable on the device being used.

Given \( x, y \) (& \( z \) is implied) the R, G, B can be found from the equation:

\[
\begin{bmatrix}
x_r & x_g & x_b \\
y_r & y_g & y_b \\
z_r & z_g & z_b \\
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix}
\]
Color Science — Out-of-Gamut colors [2]

- The color is out-of-gamut, if from the previous equation, any of R, G or B turns negative
- Approximate with the closest in-gamut color
Color Science — CIELAB Color Model

- CIELAB is three-dimensional
- Accurately represents the steps between colors
- In CIELAB,
  - One axis (a*) plots values between red and green;
  - Second axis plots between blue and yellow (b*); while the
  - Third axis plots the lightness or luminance (L*) from white to black
Color Science – CIELAB Color Model [2]

- CIELAB space quantifies the differences in the perceived color & brightness
- CIELAB offers a reference color space within which particular color gamuts can be compared.
  - Values from a particular gamut can be re-encoded as CIELAB values and then other devices can take those values and covert them into their own color gamut.
  - CIELAB is a better solution, mathematically or computationally, comparing to CIEXYZ
The CIEXYZ to CIELAB transformation is

\[ L^* = 116 \left( \frac{Y}{Y_n} \right)^{(1/3)} - 16 \]

\[ a^* = 500 \left[ \left( \frac{X}{X_n} \right)^{(1/3)} - \left( \frac{Y}{Y_n} \right)^{(1/3)} \right] \]

\[ b^* = 200 \left[ \left( \frac{Y}{Y_n} \right)^{(1/3)} - \left( \frac{Z}{Z_n} \right)^{(1/3)} \right] \]

Where \( X_n, Y_n \) & \( Z_n \) are \( X, Y, Z \) values of white point

- Color difference Formula

\[ \Delta E = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \]
Color Science – Munsell Color Naming System

- Color space is continuous, naming of colors requires quantization
- A common color space used for naming is HSI color space. Each of the values can gradually change.
  - Gradual change of 'hue' can be described by names such as red, orange red, orange, yellowish orange, yellow, green yellow, green, sea green, cyan, blue, violet and purple.
Gradual change of saturation may for example be expressed by adding to a color name labels like gray, grayish, moderate, strong and vivid.

Gradual change of intensity may be expressed by adding labels such as 'blackish', very dark, dark, medium, light (or pale) and very light.

- Munsell Naming System uses three axes to naming colors
  - Value (black to white) (9 steps)
  - Hue (40 steps around a circle) *Radius of circle varies
  - Chroma (16 levels)
Color Science — Munsell Color Naming System [3]

- HSI is very different three-dimensional color space from RGB or CYM. The following figure illustrates a common representation of this space. The cone shape has one central axis representing intensity. Along this axis are all the gray values, with black at the pointed end of the cone and white at its base. The greater the distance along this line from the pointed end, or origin, the brighter or higher the intensity.

- If this cone is viewed from above, it becomes a circle. Different colors, or hues, are arranged around this circle - the familiar color wheel used by artists. Hues are determined by their angular location on this wheel. Saturation, or the richness of color, is defined as the distance perpendicular to the intensity axis. Colors near the central axis have low saturation and look pastel. Colors near the surface of the cone have high saturation.
Color Science — Other Color Coordinate Systems

- CMY (Cyan-Magenta-Yellow)
- HSL (Hue-Saturation-Lightness)
- HSV (Hue-Saturation-Value)
- HSI (Hue-Saturation-Intensity)
- HCI (Hue-Chroma-Intensity)
- HVC (Hue-Value-Intensity)
- HSD (Hue-Saturation-Darkness)
Color Models for Image – RGB Vs CMY

- Additive Vs Subtractive Models
- Additive model
  - Used in computer displays, Uses light to display color, Colors result from transmitted light
  - Red+Green+Blue=White
- Subtractive Models
  - Used in printed materials, Uses ink to display color, Colors result from reflected light
  - Cyan+Magenta+Yellow=Black

Additive color: Light
RGB primaries; CRT monitors

Subtractive color: Ink
CMY primaries; Film, prints

RGB & CMY Cubes

- Conversion From RGB to CMY

\[
\begin{bmatrix}
C \\
M \\
Y
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} - \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

- Conversion From CMY to RGB

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} - \begin{bmatrix}
C \\
M \\
Y
\end{bmatrix}
\]
Eliminating amounts of yellow, magenta, and cyan that would have added to a dark neutral (black) and replacing them with black ink.

Four-color printing uses black ink (K) in addition to the subtractive primaries yellow, magenta, and cyan.

Reasons for Black addition includes:
- CMY mixture rarely produces pure black.
- Text is typically printed in black and includes fine detail.
- Cost saving: Unit amount of black ink rather than three unit amounts of CMY.

- Used especially in the printing of images

![CMYK Colors](image)

**CYAN**  **MAGENTA**  **YELLOW**  **BLACK**

**FINAL CMYK**

**DETAIL VIEW**
Color Models In Video – YUV Model

- If \( R' = \frac{1}{R_Y} \), \( G' = \frac{1}{G_Y} \) and \( B' = \frac{1}{B_Y} \)
- We define
- \( Y' = 0.299 \, R' + 0.587 \, G' + 0.114 \, B' \)
- \( U = B' - Y' \)
- \( V = R' - Y' \), with

\[
\begin{bmatrix}
Y' \\
U \\
V
\end{bmatrix} = \begin{bmatrix}
0.299 & 0.587 & 0.144 \\
-0.299 & -0.587 & 0.886 \\
0.701 & -0.587 & -0.114
\end{bmatrix} \begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix}
\]
Color Models In Video – YIQ Model

- Y’ is the same.
- \( I = 0.877283 (R' - Y') \cos 33^\circ - 0.492111 (B' - Y') \sin 33^\circ \)
- \( Q = 0.877283 (R' - Y') \sin 33^\circ + 0.492111 (B' - Y') \cos 33^\circ \)

\[
\begin{bmatrix}
Y' \\
I \\
Q
\end{bmatrix} =
\begin{bmatrix}
0.299 & 0.587 & 0.144 \\
0.595879 & -0.274133 & -0.321746 \\
0.211205 & -0.523083 & 0.311878
\end{bmatrix}
\begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix}
\]
Color Models In Video – YCbCr Model

- Y’ is the same.
- \( C_b = \frac{(B' - Y')} {1.772} + 0.5 \)
- \( C_r = \frac{(R' - Y')} {1.402} + 0.5 \)
- Therefore

\[
\begin{bmatrix}
Y' \\
C_b \\
C_r
\end{bmatrix} =
\begin{bmatrix}
0.299 & 0.587 & 0.144 \\
-0.168736 & -0.331264 & 0.5 \\
0.5 & -0.418688 & -0.081312
\end{bmatrix}
\begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix} +
\begin{bmatrix}
0 \\
0.5 \\
0.5
\end{bmatrix}
\]
Reference
Chapter 4
Chapter 15 of D. Hearn & M. P. Baker
Computer Graphics