Texture Classification in Tangent Space: A Hybrid IMM/SVM Approach
Outline

- Wavelet domain probabilistic model based method for texture image classification
- Fisher kernel method
- The hybrid IMM/SVM system proposed in this work
- Experimental results
Wavelet Domain Probabilistic Model Based Method for Texture Image Classification

- Texture image in wavelet domain:
Three inter-coefficient dependencies in wavelet domain:

- Intra-subband dependency
  - **Non-Gaussiannity**: marginal distributions of coefficients in each subband is in peaky, heavy tailed shape.
  - **Clustering**: coefficients of each subband display strong adjacent (local) dependency among each other.

- Inter-subband dependency at the same scale
  - **Persistence**: amplitudes of wavelet coefficients tend to propagate across subbands.

- Inter-scale dependency along the same orientation
  - **Persistence**: amplitudes of wavelet coefficients tend to propagate across scales.
Intra-subband dependency: histogram of each subband:
Inter-subband, inter-scale dependencies:
Wavelet-domain probabilistic model based method for texture classification:

- Probabilistic models previously proposed:
  - Generalized Gaussian density (GGD);
  - Independent Mixture Models (IMM);
  - Independent HMMs (IHMM);
  - Hidden Markov Tree (HMT);
  - Hidden Markov Tree – 3 subbands (HMT-3S).
Inter-coefficient dependencies utilized by various wavelet-domain probabilistic models:

<table>
<thead>
<tr>
<th>Model</th>
<th>IntraSub</th>
<th>SSInterSub</th>
<th>SSubInterS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-Gaussian</td>
<td>Clustering</td>
<td>Persistence</td>
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<tr>
<td>GGD</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>IMM</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>IHMM</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>HMT</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>HMT-3S</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Fisher Kernel Method

- Fisher scores: representing the influence of the model parameters on the generation of the observation.
  - Partial derivatives of log likelihood with regard to model parameters.
    \[
    \nabla_{\hat{\theta}}(O) = \left[ \frac{\partial \log f(O|\theta)}{\partial \theta_1}, \ldots, \frac{\partial \log f(O|\theta)}{\partial \theta_p} \right]^T |_{\theta = \hat{\theta}}
    \]
    - Residing in the tangent space of a Riemannian manifold with a local metric scaled by the *Fisher information matrix*:
      \[
      I(\hat{\theta}) = E_O(\nabla_{\hat{\theta}}(O)\nabla_{\hat{\theta}}(O)^T)
      \]
    - Similarity of two observations with regard to \( \hat{\theta} \):
      \[
      K_{\hat{\theta}}(O_i, O_j) = \nabla_{\hat{\theta}}(O_i)^T I(\hat{\theta})^{-1} \nabla_{\hat{\theta}}(O_j)
      \]
Fisher scores illustrated:

- **Gaussian**: \( p(x) \sim \mathcal{N}(\mu, \sigma^2) \)

\[
\nabla_{\mathcal{N}}(x) = \begin{bmatrix} \nabla^{\mu}(x) \\ \nabla^{\sigma}(x) \end{bmatrix} = \begin{bmatrix} \frac{x-\mu}{\sigma^2} \\ \frac{(x-\mu)^2}{\sigma^3} - \frac{1}{\sigma} \end{bmatrix}.
\]

- **HMM**: \( \lambda = (A, B, \pi) \)

\[
\nabla_{\lambda}(O) = \begin{bmatrix} \nabla^{A}(O) \\ \nabla^{B}(O) \\ \nabla^{\pi}(O) \end{bmatrix} = \ldots.
\]
Roles played by the training data set:
  - Estimate the parameters of probabilistic models for each class.
  - Estimate the parameters of classifiers (e.g. SVM) based on discriminative training.

Role played by the probabilistic models:
  - A feature vector generator (rather than a classifier).
A Hybrid IMM/SVM System for Texture Classification

- Probabilistic models chosen in our scheme: IMM
  - Mathematical representation:
    \[ \theta_{\text{IMM}} = \{\pi_{l,b,m}, \mu_{l,b,m}, \sigma_{l,b,m}\}, \]
    where \( \pi_{l,b,m}, \mu_{l,b,m}, \) and \( \sigma_{l,b,m} \) are mixing coefficient, mean and variance of a Gaussian mixture model (GMM).
  - Rationale:
    - IMM can capture most prevailing statistical texture features in wavelet domain – it’s performance is comparative to (not substantially lower than) that of more sophisticated model structures such as HMT and HMT-3S.
    - The computational complexity of fisher scores from IMM is much lower than that from other more sophisticated probabilistic models.
Computing Fisher scores on IMM:

\[
\nabla \pi_{\bar{l}, \bar{b}, \bar{m}}(w) = \sum_{i, j=1}^{N_{\bar{l}}} \frac{g(w_{i, j}^{\bar{l}, \bar{b}} | \mu_{\bar{l}, \bar{b}, \bar{m}}, \sigma_{\bar{l}, \bar{b}, \bar{m}})}{\sum_{m=1}^{M} \pi_{\bar{l}, \bar{b}, m} g(w_{i, j}^{\bar{l}, \bar{b}} | \mu_{\bar{l}, \bar{b}, m}, \sigma_{\bar{l}, \bar{b}, m})},
\]
\[
\bar{l} = 1, ..., L, \quad \bar{b} = 1, 2, 3, \quad \bar{m} = 1, ..., M.
\]

\[
\nabla \mu_{i, b, m}(w)
\]
\[
= \sum_{i, j=1}^{N_{\bar{l}}} \frac{\pi_{\bar{l}, \bar{b}, \bar{m}} g(w_{i, j}^{\bar{l}, \bar{b}} | \mu_{\bar{l}, \bar{b}, \bar{m}}, \sigma_{\bar{l}, \bar{b}, \bar{m}})}{\sum_{m=1}^{M} \pi_{\bar{l}, \bar{b}, m} g(w_{i, j}^{\bar{l}, \bar{b}} | \mu_{\bar{l}, \bar{b}, m}, \sigma_{\bar{l}, \bar{b}, m})} \times \frac{w_{i, j}^{\bar{l}, \bar{b}} - \mu_{\bar{l}, \bar{b}, \bar{m}}}{(\sigma_{\bar{l}, \bar{b}, \bar{m}})^2},
\]
\[
\bar{l} = 1, ..., L, \quad \bar{b} = 1, 2, 3, \quad \bar{m} = 1, ..., M.
\]

\[
\nabla \sigma_{i, b, m}(w)
\]
\[
= \sum_{i, j=1}^{N_{\bar{l}}} \frac{\pi_{\bar{l}, \bar{b}, \bar{m}} g(w_{i, j}^{\bar{l}, \bar{b}} | \mu_{\bar{l}, \bar{b}, \bar{m}}, \sigma_{\bar{l}, \bar{b}, \bar{m}})}{\sum_{m=1}^{M} \pi_{\bar{l}, \bar{b}, m} g(w_{i, j}^{\bar{l}, \bar{b}} | \mu_{\bar{l}, \bar{b}, m}, \sigma_{\bar{l}, \bar{b}, m})} \times \left[ \frac{(w_{i, j}^{\bar{l}, \bar{b}} - \mu_{\bar{l}, \bar{b}, \bar{m}})^2}{(\sigma_{\bar{l}, \bar{b}, \bar{m}})^3} - \frac{1}{\sigma_{\bar{l}, \bar{b}, \bar{m}}} \right],
\]
\[
\bar{l} = 1, ..., L, \quad \bar{b} = 1, 2, 3, \quad \bar{m} = 1, ..., M.
\]
- Clustering property illustrated:

<table>
<thead>
<tr>
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<tbody>
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<td>L1.H.pi1</td>
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<td><img src="L1.H.pi2" alt="Image" /></td>
<td><img src="L1.V.pi1" alt="Image" /></td>
<td><img src="L1.V.pi2" alt="Image" /></td>
<td><img src="L1.D.pi1" alt="Image" /></td>
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<td><img src="L1.H.pi1" alt="Image" /></td>
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<td><img src="L1.D.pi1" alt="Image" /></td>
<td><img src="L1.D.pi2" alt="Image" /></td>
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</table>
Overview of the proposed system:

Feature Vector Generation

Training

Classification
Experimental Results

- Brodatz texture database:
Benchmark methods:
- Energy Signature (ES): \( E_{l,b} = \frac{1}{N_l^2} \sum_{i,j=1}^{N_l} (w_{i,j}^{l,b})^2 \)
- ES/SVM
- IMM

Proposed method:
- IMM/SVM
Recognition rates:

<table>
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<th>Training images</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
<tr>
<td>ES</td>
<td>0.4218</td>
<td>0.4459</td>
<td>0.4474</td>
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<td>ES/SVM</td>
<td>0.5324</td>
<td>0.5684</td>
<td>0.5235</td>
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<td>IMM</td>
<td>0.5430</td>
<td>0.5685</td>
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<td>IMM/SVM</td>
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<tr>
<td>$\nabla_{\pi}(w)$</td>
<td>0.7658</td>
<td>0.8090</td>
<td>0.8488</td>
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<td>$\nabla_{\mu}(w)$</td>
<td>0.0794</td>
<td>0.1063</td>
<td>0.1321</td>
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<td>$\nabla_{\sigma}(w)$</td>
<td>0.7133</td>
<td>0.7883</td>
<td>0.8248</td>
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<tr>
<td>$\nabla_{\theta_{IMM}}(w)$</td>
<td>0.7838</td>
<td>0.8234</td>
<td>0.8604</td>
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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>ES</td>
<td>0.4550</td>
<td>0.4710</td>
<td>0.4760</td>
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<td>0.5417</td>
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<td>IMM</td>
<td>0.5248</td>
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<tr>
<td>IMM/SVM</td>
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<tr>
<td>$\nabla_{\pi}(w)$</td>
<td>0.8581</td>
<td>0.8752</td>
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<tr>
<td>$\nabla_{\mu}(w)$</td>
<td>0.1610</td>
<td>0.1802</td>
<td>0.2012</td>
</tr>
<tr>
<td>$\nabla_{\sigma}(w)$</td>
<td>0.8491</td>
<td>0.8687</td>
<td>0.8964</td>
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<tr>
<td>$\nabla_{\theta_{IMM}}(w)$</td>
<td>0.8839</td>
<td>0.8906</td>
<td>0.9069</td>
</tr>
</tbody>
</table>
Recognition rates (Cont.):

- ES
- ES/SVM
- IMM
- IMM/SVM-PI
- IMM/SVM-MU
- IMM/SVM-SIGMA
- IMM/SVM-IMM
Thank You!

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