NASA Terrestrial Ecology Award NNX09AL03G
Mapping Changes in Shrub Abundance and Biomass in Arctic Tundra using NASA Earth Observing System Data: A Structural Approach

Summary of Research, March 2015

Principal Investigator
Mark Chopping
Department of Earth and Environmental Studies
Montclair State University, 1 Normal Ave, Montclair, NJ 07043
E-mail: chopping@pegasus.montclair.edu  Tel.: (973) 655-7384  Fax: (973) 655-4072

Co-Investigators
Crystal B. Schaaf  Univ. Mass.-Boston  crystal.schaaf@umb.edu  (617) 287-7440
Ken Tape  Univ. Alaska-Fairbanks  kdtape@alaska.edu  (907) 474-7183

Collaborators
Alan H. Strahler  Boston University  alan@bu.edu  (617) 353-5984
W. Gareth Rees  SPRI/Cambridge  wgr2@cam.ac.uk  +44 (1223) 336540
David J. Selkowitz  USGS AK Sci. Ctr.  dselkowitz@usgs.gov  (907) 786-7146

Post-Doctoral Researcher & Graduate Student
Zhuosen Wang  NASA Goddard Space Flight Center  zhuosen.wang@nasa.gov
Rocio Duchesne  University of Wisconsin-Whitewater  duchesnr@uww.edu

Overview

The goal of this project was to map shrub abundance and woody biomass in Arctic tundra using NASA moderate resolution solar wavelength bidirectional reflectance factor (BRF) imagery over the Arctic tundra biome by exploiting the structural information in these data using multi-angle remote sensing methods. The tasks required for this project thus divided into three categories: acquisition of reference data; development of techniques to exploit NASA Earth Observing System MISR and MODIS data sets via BRDF and geometric-optical model inversion and spectral-angular metrics; and implementation for mapping over large areas (e.g., Arctic tundra).

Remote sensing of shrubs in Arctic tundra with moderate resolution MISR and MODIS reflectance data is challenging: shrubs are dark in the red wavelengths (BRF < 0.05 for alder leaves), the non-shrub grass matrix has a similar spectral reflectance (BRF = ~0.065), and illumination is weaker than at lower latitudes with long atmospheric paths increasing uncertainty in surface BRF estimates that the mapping ultimately depends upon. Additional challenges are posed by the heterogeneity of landscapes, including talus slopes and high variation in surface water, with some
landscapes marked by numerous ponds or water tracks. Attempts to map shrubs using spectral data have until recently focused on the use of the Normalized Difference Vegetation Index, an approach that has been shown from first principles to be somewhat compromised (e.g., Rees et al. 1998). Beck et al. (2011) used Landsat imagery with a Random Forest regression algorithm to produce the first circa 2000 “baseline” maps of fractional cover for all shrubs and tall shrubs (height > 1 m) for the North Slope of Alaska. This approach benefits from the much smaller ground-projected field-of-view (30 m) and consequently higher cover values and larger in-pixel signal than available from MISR or MODIS (>= 250 m). However, the Landsat-based maps are subject to obvious anomalies (e.g., missing data, disjuncture’s, and spurious shrub predictions in the Canning River delta) and have low correlations with our shrub cover reference data (intermediate database, N=418). Since our Arctic shrub mapping project exploits the effects of physical shrub canopy structure on NASA Earth Observing System MISR and MODIS reflectance data, the focus was on estimating tall shrub – rather than all shrub – abundance.

Summary of Findings

In this project we worked first to construct a robust shrub canopy reference database for an extensive array of sites; test techniques for exploiting MISR and MODIS data via BRDF and geometric-optical model inversion, multi-angle metrics, and empirical methods; and implement mapping over the N. Slope of Alaska and subsequently the entire Arctic tundra biome. The project’s key outputs and findings are:

- Reference data are key to mapping and the only means of evaluating mapping results; we expended important efforts on the construction of an unique, robust, and extensive validation database that has been published as a data set of the North American Carbon Program (see the attached metadata; the data are at http://daac.ornl.gov/cgi-bin/dsviewer.pl?ds_id=1270).
- For mapping shrubs over large areas, we selected a Boosted Regression Tree (BRT) prediction method that can exploit angular metrics such as BRDF model kernel weights to provide more accurate shrub cover estimates, having previously assessed geometric-optical (GO) model inversion with MISR and MODIS data, the MODIS structural scattering index (SSI) metric, artificial neural network, and Bootstrap Forest approaches. We selected MISR kernel weights for the mapping, as these are less noisy than the corresponding MODIS weights that are based on BRF data accumulated over many days.
- Geometric-optical model inversion for mapping tall shrubs over the tussock grass, lichen, rock, and dwarf shrub surfaces of Alaskan Arctic tundra is compromised by the difficulty of predicting the BRDF of the non-tall-shrub background (and the BRDF model volume scattering kernel weight in particular); and by the low contrast with erect shrubs, although it has utility elsewhere.
- At MODIS and MISR scales (~250 m) tall shrub fractional cover is low even in the most dense scenarios, with a mean of 0.06 (maximum = −0.29). This represents a major challenge for tracking shrub cover changes because the signal is small and the range narrow. The low precision of cover estimates means that it is difficult to reliably determine changes in shrub cover over the 2000-2010 period.
- Mapping trees at the tundra-taiga interface in Russia and shrubs and trees in N. Canada can be performed with good accuracy using GO model inversion against MISR red band multi-angle BRFs, providing maps of canopy cover and height, while spectral measures can be deceptive, with larger NDVI for grass or crops vs forest.
### Significant Milestones

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 2009</td>
<td>Official start of project, engagement of PhD student at Boston University.</td>
</tr>
<tr>
<td>Aug 2009</td>
<td>First field campaign on the Chandler River, following aerial survey.</td>
</tr>
<tr>
<td>Jan 2010</td>
<td>Engagement of PhD student at Montclair State University</td>
</tr>
<tr>
<td></td>
<td>Development of the CANAPI algorithm for crown detection.</td>
</tr>
<tr>
<td></td>
<td>First experiments with MISR GO models for tundra tall shrub mapping.</td>
</tr>
<tr>
<td>July 2010</td>
<td>Second field campaign on the Chandler River.</td>
</tr>
<tr>
<td>May 2011</td>
<td>Development of new, fast version of AMBRALS with WoD output.</td>
</tr>
<tr>
<td></td>
<td>First tundra-taiga mapping efforts with a MISR/GO approach.</td>
</tr>
<tr>
<td>July 2011</td>
<td>Third field campaign, on the Dalton Highway.</td>
</tr>
<tr>
<td>Feb 2012</td>
<td>Experiments with MODIS BRDF and GO models for shrub mapping.</td>
</tr>
<tr>
<td>2012-2013</td>
<td>Assessment of shrub cover mapping methods (GO model, SSI, and empirical methods: ANN, bootstrap forest, boosted regression trees). Reference database size increased to more than 1,000 sites on N. Slope of Alaska. Second tundra-taiga mapping effort with a MISR/GO approach.</td>
</tr>
<tr>
<td>Mar 2014</td>
<td>Work on production of shrub maps for Canada and parts of Russia.</td>
</tr>
<tr>
<td></td>
<td>Completion of the reference database and writing up the metadata.</td>
</tr>
<tr>
<td>Aug 2014</td>
<td>Production of MISR-based maps of shrub abundance across the Arctic tundra biome for Alaska and parts of Canada, Scandinavia, and Russia using BRT and GO models; testing of compositing algorithms and screening criteria.</td>
</tr>
<tr>
<td>Mar 2015</td>
<td>Publication of NACP Data Set “Woody Vegetation Characteristics of 1,039 Sites across the North Slope, Alaska”; evaluation of N. Canada maps with NGA/CDA imagery; summary of research submitted.</td>
</tr>
</tbody>
</table>

### Significant Outputs

- Extensive, robust, shrub cover reference database for the North Slope of Alaska published as the North American Carbon Program Data Set “Woody Vegetation Characteristics of 1,039 Sites across the North Slope, Alaska”, at the Oak Ridge National Laboratory Distributed Active Archive Center, URL: http://daac.ornl.gov/cgi-bin/dsvviewer.pl?ds_id=1270.
- Shrub/tree maps for parts of Canada, Scandinavia, & Russia, 2010 (MISR/BRT; MISR/GO).
- Four papers published, one submitted, one currently in preparation; three more anticipated.
- New, fast version of AMBRALS algorithm with Weights of Determination (Zhuosen Wang).
- Robust surface water mapping based on NIR BRF, RTLS-R RMSE, and GO model cover.

### Research Activities in 2014 - 2015

We continued to work on refinement of the BRT model for mapping shrub cover on the North Slope of Alaska with a view to improving precision, as this is critical to the robust assessment of change over short periods (11 years). The PI was incapacitated for almost 2 months by a broken clavicle in May 2014 owing to a traffic accident; however, by late summer of 2014 MISR-based maps of shrub abundance for parts of Canada, Scandinavia, and Russia, were made, with much effort expended on the testing of compositing and screening algorithms as imagery for this region has much cloud, surface water and non-vegetated land that must be excluded from map outputs.
A shrub cover map was made for the N. Canada using the same BRT model developed for Alaska but with fewer predictors: nadir camera Blue, Green, Red, and NIR BRFs; RossThick-LiSparseMODIS-Reciprocal (RTLS-R) BRDF model kernel weights, model-fitting root mean square error (RMSE), and Normalized BRDF-Adjusted Reflectance with a solar zenith angle of 45° (NBAR45). A simple min(RMSE) compositing criterion was used, resulting in a map with few gaps (though data for other years would be needed for wall-wall mapping; Figure 15-1(a)). Earlier compositing work had used near-infrared BRF, extreme outliers, topographic shading, and cloud flags but this was too severe; and water and shaded areas and outliers can be screened out a posteriori. In the course of producing the shrub map for N. Canada the need for a more robust method for excluding observations that include surface water was recognized. The method usually adopted to do this in remote sensing studies is to set a threshold based on the near-infrared (NIR) BRF value. However, it is hard to establish a threshold that separates water from land reliably; setting a NIR threshold that detects small water bodies or those that occupy a fraction of a 250 m cell (<0.17) results in large parts of the map being screened out (Figure 15-1(b)). The first attempt to solve this problem used the RTLS-R BRDF model geometric scattering kernel weight (geo) that is highly sensitive to surface scattering and therefore structure, with relatively low dependence on brightness. It is therefore able to screen for water reliably and without excluding vast swathes of the map that have valid predictions (Figure 15-1(c)(d)). However, forest in the Mackenzie River Delta was also screened out by this method; a better solution was based on a less stringent NIR BRF threshold (< 0.1), BRDF model RMSE > 0.045, and GO model fractional cover > 0.45 (Figure 15-2).
Figure 15-2. A section of the Canada NWT maps from GO model inversion showing the Mackenzie River delta in the northwestern part (a) fractional cover (b-w: 0.0-0.5) (b) mean canopy height (b-w: 0 - 4 m). Gray indicates bad input data; blue indicates water/cloud/non-vegetated pixels flagged using MISR NIR BRF < 0.1, RTLS-R BRDF model RMSE > 0.045, and GO model fractional cover > 0.45. The map layers are available at http://csam.montclair.edu/~chopping/tundra/can_go.php.
The MISR/BRT N. Canada shrub cover map showed broad agreement with imagery but visual evaluation against QuickBird panchromatic imagery from the National Geospatial-Intelligence Agency (NGA) Commercial Archive Data was not encouraging; the BRT model developed for Alaska does not seem to be generalizable to N. Canada. A further mapping effort was therefore made for N. Canada using inversion of the Simple Geometric-optical Model (SGM) against MISR red band BRFs, as for the earlier Kola Peninsula mapping. The contribution of the background at the various sun-target-view angles provided by the RTLS-R BRDF model, estimated a priori using the kernel weights of the same mode, with regression equations calibrated using the same estimates obtained over the Kola Peninsula (from interpretation of 53 high spatial resolution image chips extracted from Google Earth with CANAPI; see page 28 and Figure 20). For these GO model inversions, the mean crown radius ($r$) and tree number density ($\lambda$, trees per square meter) were adjusted by minimizing an absolute root mean square error (RMSE) cost function using the Praxis optimization algorithm. This allowed retrieval of fractional crown cover and mean canopy height ($h+b$, where $h$ is mean crown center height and $b$ is mean crown vertical radius).

The maps obtained via GO model inversion appear to be reasonable, with both cover and height clearly related to shrub and tree density (Figure 15-3; cover only is shown but height follows a similar distribution). This verifies that the structural signal in MISR imagery can be exploited to map tall woody plants in tundra and taiga, though more work is required for comprehensive validation. The MISR/GO (and MISR/BRT) maps also point to the need for careful screening for cloud, water, non-vegetated surfaces, and missing data; efficient compositing methods; and to combine data for more than one growing season in order to produce maps without large gaps.

A new map for the northern part of Alaska was also made in early 2015, using a new BRT model driven with MISR and ancillary topographic and geographic variables. The model was calibrated
using the NACP Woody Plant reference data set Duchesne et al. (2015a-b). For this model the target variable was set to arcsine(fractional_cover), learning rate was set to 0.005 and Tree Complexity was set to 3. This yielded an $R^2$ of 0.57 vs. the extensive reference data (N=1,039).

Figure 15-4 New map of tall shrub cover in 2000/2001 for Northern part of Alaska from a new BRT model driven with MISR and other variables, calibrated using the NACP reference data set (see Table 1).

Figure 15-5 (a) and (c) sections of the new Alaska shrub cover map (b) and (d) corresponding Google Earth imagery.
The tasks envisaged for this period were to evaluate means for increasing mapping precision using our extended reference database; expand the mapped area to cover the Arctic; explore the potential for mapping the tundra-taiga interface; fill gaps to produce wall-to-wall pan-Arctic maps; and compare the trajectory of shrub abundance with that of summer albedo. We focused on expansion and enhancement of the reference database, extension of mapping to the pan-Arctic, and methods for mapping trees at the tundra-taiga interface; while gap-filling methods were investigated in early 2014. Comparisons of changes in shrub cover and with those in albedo have not been effected as the precision in the change estimates available is insufficient.

Research Activities, April – October 2012

We used the reference data on shrub cover and height collected at field sites on the North Slope of Alaska in 2010 and 2011 and used these to calibrate semi-automated methods that exploit a version of the Canopy Analysis with Panchromatic Imagery (CANAPI) algorithm (Chopping, 2011) adapted for shrubs in Arctic tundra. This allowed us to generate a larger reference database, using high spatial resolution (0.6 m) QuickBird panchromatic imagery. The intermediate reference database spanned 418 sites of 250 m on a side, corresponding to the grid onto which MISR red band multi-angle imagery is mapped. We checked and tested the validation data, comparing field- and imagery-based estimates, and corrected the latter for a unit error (pixels vs. meters) in earlier estimates. We also recalculated and applied new adjustment factors to the CANAPI-based estimates and used these new data to recalibrate the background BRDF kernel weight predictions used in our GO model inversion protocol.

We worked on evaluation of several multi-angle remote sensing methods for large scale mapping over Alaskan Arctic tundra (MISR, MODIS) and the tundra-taiga transition zone in the Kola Peninsula (MISR). Over the summer of 2012 efforts were focused on checking our validation database and applying these results to GO model inversion work (MISR, MODIS), exploitation of BRDF model kernel weights in predicting tall shrub cover (MISR), and comparisons with the Landsat-derived “circa 2000 baseline” total and tall shrub maps (Beck et al. 2011). In particular, we tested the use of alternative background BRDF prediction variables in GO model inversions, including spectral measures (NDVI) and “reduced” BRDF models with only two kernels (iso+vol and iso+geo), since we had shown that retrieval accuracy was owing the lack of precision in predicting the volume-scattering kernel weight. These attempts resulted in accurate retrievals when optimal background BRDFs were used – but again, poor correlations when regression equations were used to predict background (non-shrub) BRDFs.

Efforts to exploit the isotropic, volume scattering, and geometric scattering kernel weights (retrieved via inversion of the RTLS-R BRDF model against MISR red band BRFs mapped at 250 m) have been more promising, especially when these metrics are combined with nadir camera BRFs, as reported below. The project’s close date was extended in order to allow us time to fully evaluate the potential for increasing precision in mapping tall shrubs using our extended reference database; expand the mapped area and fill gaps to produce wall-to-wall pan-Arctic maps; explore the potential for mapping the tundra-taiga interface; and compare the trajectory of shrub abundance with that of summer albedo.
A reference database with tall shrub fractional cover estimates for over 1,000 250 x 250 m sites has been made available to the community through the North American Carbon Program/Oak Ridge National Laboratory in early 2015 (see the attached metadata document). The database was constructed following aerial survey, field inventory (2010, 2011), and computer-aided interpretation of large volumes of QuickBird and GeoEye high resolution panchromatic imagery. Estimates of shrub fractional cover, mean crown radius, and mean shrub height were collected at 24 field sites following a belt-transect method; these allow the image-derived estimates to be calibrated. Structural information was obtained for the field sampled sites using ImageJ and CANAPI. CANAPI identifies and delineates shrub crowns and the length of the shadow cast by them (Chopping, 2011). The macro can be used to estimate mean shrub height, mean crown radius, total number of shrubs, and fractional cover (Figure 1). CANAPI-derived estimates were adjusted to the field estimates using linear equations (in situ field measurements are always considered more reliable).

In the summer of 2013 the database was expanded and enhanced to obtain cover estimates for additional sites that are both geographically distant and ecologically distinct from those already included. We were able to do this thanks to the availability of large volumes of high resolution imagery from the National Geospatial-Intelligence Agency (NGA) Commercial Archive Data (http://cad4nasa.gsfc.nasa.gov/) that provides access for NASA Earth Science Investigators. The additional 250 x 250 m sites were selected from 20 panchromatic QuickBird and GeoEye scenes and were chosen to include locations within four broad physiognomic categories and across the broad latitudinal and longitudinal range of the North Slope of Alaska. The CANAPI algorithm was tuned for each subset and estimates were adjusted using regression coefficients derived previously.

Figure 1. Panchromatic QuickBird subsets for two sites. Circles indicate CANAPI-identified shrub crowns. In (a) shrubs are sparsely distributed but CANAPI identifies almost all of them. In (b) shrubs are arranged in thickets along the water track; although CANAPI is unable to isolate individual shrubs it is still able to map shrub cover.
Intermediate Reference Database & Calibration/Validation with Field Data

To establish an extensive database based on field measurements of shrub canopy density, cover and height, a field campaign was carried out on the North Slope of Alaska during summer, 2010 (July 24th–August 12th). A crew of three people descended the Chandler and Colville Rivers using inflatable boats from the Brooks Range (68°45′06.4″N, 152°18′35.8″W) to the Arctic Coastal Plain (69°40′01.6″N, 151°30′29.7″W). Sampling sites fell in an altitudinal and climatic gradient, with the southernmost sites at higher elevation (~280 m a.m.s.l.) and influenced by the continental climate coming from the Brooks Range; while the northernmost sites were located at much lower elevations (~95 m a.m.s.l.), where maritime climate conditions prevail.

Structural data for the woody Arctic vegetation were collected at fourteen sites located within 1 km of a river. Each sampling site had an area of 250 × 250 m which corresponds to the mapped MISR cell (projected). The sites were selected in advance using aerial photographs and high resolution imagery (QuickBird and IKONOS) and represent a variety of vegetation types and conditions. The sampling period was in the peak growing season; therefore no phenological changes were observed in the vegetation at that time.

The belt transect method was used to survey 13 out of the 14 sites. At site 14th, all shrubs were sampled as they were so few. Transects were 5 m wide and 250 m long and the number of transects per site varied between 5 or 10 depending on the difficulty to access the area (Figure 2(a)). The geographic location was recorded and a photographic record of each surveyed shrub was taken and used to estimate width (horizontal extent) and height (vertical extent from base to top of foliage) (Figure 2(b)). These estimates allow us to determine fractional cover, mean crown radius, and mean shrub height for the entire 250 × 250 m area of each site.

![Figure 2](image1.png)

Figure 2. (a) Arrangement of the belt transects at a site with five transects. The transects were laid out according to the altitudinal gradient of the terrain. (b) Field picture at sampling site collecting photographic record of shrubs. Next to the shrub is a vertical 2 m scaled rod used later to calibrate crown width and height of the shrub.

Enlargement of the structural database was pursued by exploiting the CANAPI algorithm in conjunction with five QuickBird panchromatic scenes of about 27.5 km² each that covered 10 out of the 14 sampling sites (Figure 3). For the other four sampling sites it was not possible to use CANAPI to measure shrub cover and height because the available IKONOS imagery was not suitable. Our field data are considered to be very reliable because each shrub was physically measured in situ. CANAPI is a user-tunable algorithm that uses high resolution panchromatic...
imagery to identify and measure tree or shrub crowns and heights by estimating the length of the shadows cast (Chopping, 2011). However, in this environment CANAPI estimates demonstrate some systematic errors. CANAPI outputs the crown radius and area for all detected shrubs; and the height of each shrub that casts a shadow that is not truncated by another crown or the edge of the image. These estimates were used to estimate mean shrub height, mean crown radius, total number of shrubs, and fractional cover for each site.

Figure 3. Google Earth representation of part of Alaska above the Brooks Range between 68.7° N and 69.7° N, displaying the locations of the QuickBird images used in reference data work as red rectangles. The reference database was constructed using field data collected at field sites within these areas and extended to 418 250 × 250 m sites.

The construction of the intermediate reference database of 418 sites proceeded in two steps: the derivation of linear equations for adjustment of the CANAPI-derived estimates for the field sites so that they more closely match (field data are considered more reliable); followed by the application of these equations with the outputs of CANAPI runs for a much larger number of sites.
The first step toward the extension of the reference database was to derive CANAPI estimates for the 10 sampling sites that had field data associated with it, and to compare both estimates. The difference between field estimates and the CANAPI estimates for fractional cover was analyzed in conjunction with the QuickBird chips for the 10 sampling sites. This revealed that as the number of shrub clusters increased (i.e., thickets), CANAPI underestimated fractional cover because it was unable to break down the clusters into individual shrubs. Where vegetation was sparse, CANAPI estimates were very close to field data. Similar behavior occurred for total number of shrubs estimates. On the other hand, mean crown radius and mean height CANAPI estimates were always lower in comparison with the field estimates. In the case of mean crown radius this difference (always less than 0.3 m) seems to be a systematic error of CANAPI probably due to pixel resolution (panchromatic QuickBird resolution is 0.6 m). As for mean height, the difference between CANAPI and field estimates might be due to the fact that CANAPI did not capture the entire length of the shadows cast by shrubs.

Figure 4. Scatter plots showing relationships between CANAPI and field estimates for 10 sampling sites. (a) correlation for fractional cover ($R^2 = 0.83$), (b) correlation for mean crown radius ($R^2 = 0.89$), (c) correlation for mean shrub height ($R^2 = 0.26$), and (d) correlation for total number of shrubs ($R^2 = 0.42$).

In order to adjust the fractional cover, mean crown radius, mean height, and total number of shrubs CANAPI estimates, regression coefficients were derived by correlating the CANAPI estimates against the corresponding field estimates via simple linear regressions, for the 10 sampling sites. Outliers were omitted from the analysis (Figure 4). High $R^2$ values for fractional cover and mean
crown radius (0.83 and 0.89 respectively) suggest that it is appropriate to use the regression coefficients to adjust CANAPI estimates. Since the regression for total number of shrubs had a medium-low $R^2$ (0.42), using the regression equation to adjust CANAPI estimates must be done with caution. In the case of mean shrub height, the $R^2$ was quite low (0.26) and it suggests that the regression coefficients are inadequate for adjusting the CANAPI estimates.

Subsequently, 418 subsets of $250 \times 250$ m – aligned with the Albers Conical Equal Area grid onto which the MISR data are mapped – were selected from the five QuickBird scenes covering our Chandler/Colville sites (Table 1). These subsets were explicitly chosen to include representatives from three geomorphologic units: floodplains, hills, and interfluves. CANAPI was run on each subset and estimates were adjusted using the regression equations previously derived. This resulted in a robust shrub cover database that includes a wide range of conditions and cover values. It has been used for training of the canopy reflectance/empirical model and validation of the retrieved fractional cover values from the model.

### Table 1. DISTRIBUTION OF SUBSETS IN THE INTERMEDIATE DATABASE ACCORDING TO LANDFORM AND LATITUDINAL GRADIENT OF THE QUICKBIRD SCENE

<table>
<thead>
<tr>
<th>QUICKBIRD SCENE</th>
<th>GEOLOGICAL FEATURE</th>
<th>NUMBER OF SUBSETS</th>
</tr>
</thead>
<tbody>
<tr>
<td>QB_20 High Latitude N69.65, W151.45</td>
<td>Hills</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>Interfluves</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Floodplains</td>
<td>35</td>
</tr>
<tr>
<td>QB_40 Mid Latitude N69.12, W151.82</td>
<td>Hills</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>Interfluves</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Floodplains</td>
<td>28</td>
</tr>
<tr>
<td>QB_50 Mid Latitude N69.04, W151.77</td>
<td>Hills</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>Interfluves</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Floodplains</td>
<td>46</td>
</tr>
<tr>
<td>QB_R1C1 &amp; QB_R1C2 Low Latitude N68.78, W152.15</td>
<td>Hills</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>Interfluves</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>Floodplains</td>
<td>30</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>418</strong></td>
</tr>
</tbody>
</table>

Analysis of the reference database reveals that fractional cover is relatively low, even in the most dense scenarios. The maximum fractional cover was 0.29 and the mean value was 0.056 (Figure 5(a)). This represents a challenge for tracking changes in shrub cover at the scale of moderate resolution remote sensing instruments because the signal is very small and the cover range is quite narrow: this will make it more difficult to determine that changes seen from year to year are owing to real changes in shrub cover and not to noise in the estimation. Mean crown radius varied between 0.46 m and 1.65 m (Figure 5(b)). The site that had the least number of shrubs still had 151 shadow-casting individuals, while the site with the most shrubs had 1598. The mean number of shrubs was 665 (Figure 5(c)).
In 2013 we expanded the reference database further by using CANAPI with sites that are distant and ecologically distinct from those included in 2012 using imager from the National Geospatial-Intelligence Agency (NGA) Commercial Archive Data site (http://cad4nasa.gsfc.nasa.gov/) that provides access to large volumes of high resolution imagery for NASA Earth Science investigators.

**MISR Surface BRF Assessment using Field Radiometry**

In order to accurately retrieve surface reflectance from MISR – which is the input data for the models used in our mapping efforts – the effect of the atmosphere (via clouds, aerosols, gases, and water vapor) must be removed. This is especially important at high latitudes where path lengths are long. The MISR Terrain data used are already atmospherically corrected by NASA; however; there may be some residual atmospheric effects that contribute to error in shrub cover estimates. Assessment of the propagation of this error is been carried out using scaled-up field-derived BRDF samples collected during the Dalton field campaign in the summer of 2011.

Nine sites were selected along the Dalton Highway, the only road that connects interior Alaska with the coastal plain. Data collection was done more than a kilometer away from the gravel road to avoid the effects of windblown dust on the vegetation. Each site had an area of 250 × 250 m and is aligned with the grid onto which the MISR Terrain data are mapped. The sites were selected in advanced using high resolution GeoEye imagery and they represent a variety of conditions and vegetation types. Unsupervised classification of multispectral high resolution images for each site was performed before the field campaign in order to locate sampling points within spectrally homogeneous areas. Initially, each site was segmented into five categories and three sampling points were selected within each category (15 locations in total); but this was found to be too difficult in the field, so the number of sampling points was reduced to seven per site (Figure 6).
The data collected at each site consisted of seven sets of multi-spectral and multi-angular photos of the target vegetation. Each set included two scans on the solar principal plane and two in the principal plane. Viewing angles resembled those from MISR ±70°, ±60°, ±45°, ±30°, ±20° degrees (forward and back-scattering direction) and nadir. Each set had a total of 44 multi-spectral multi-angular photos. Following the scans, color photos were taken at nadir view with a Canon point-and-shoot camera to facilitate identification of vegetation species in the lab. The whole process of taking the pictures for each set took between 8 to 10 minutes. Radiometric data collection was limited to the period between 10:00am and 3:00pm when the solar intensity was the highest and to keep a consistent solar zenith angle; and to times when cloud cover was less than 30%.

The instrument used for the collection of radiometric data was an ADC Tetracam camera which is a single sensor digital camera designed for capturing visible light in blue-green, red and NIR bands. The camera was mounted on a frame at 3.5 m height giving a ground resolution element of ~0.9 m at nadir which increased with view zenith angle. Reflectance panels of 2%, 5%, 10%, 20% and 40% reflectance were placed within the field of view of the camera for post-calibration of the camera DN to reflectance of the target vegetation. Calibration of the digital number was achieved for each sampling point by regressing the digital camera number against the know reflectance of the reflectance panels at nadir in the red band (Figure 7).
Figure 7. Data for site six, sampling point 12: (a) camera digital number adjustment by applying second order polynomial regression against known reflectance from reflectance panels, (b) color photograph to illustrate the arrangement of the reflectance panels within the field of view of the multispectral camera.

Regression coefficients for each sampling point were then used to derive the BRF of the target vegetation at the different viewing angles. In the case of site six, there were seven sampling points (Figure 8). BRF graphs in the red band varied depending on the composition of the target vegetation. Sampling points have different proportions of tussock, moss, cotton grass, and dwarf birch.

Although the ground reflectance measurements were taken at similar viewing angles to those of the MISR sensor, they are not exactly the same. In order to account for differences in the geometry of the viewing and illumination angles and allow valid comparisons, the RTLS-R BRDF model was adjusted against the ground bidirectional reflectance measurements using the Algorithm for Modeling Bidirectional Reflectance Anisotropies of the Land Surface (AMBRALS; Wanner et al. 1995). The retrieved isotropic scattering, volumetric scattering, and geometric scattering kernel weights describe the bidirectional reflectance distribution function (BRDF) properties of the surface. According to Brown de Colstoun et al. (1996) and Liang (2000) BRDF upscaling is largely linear. This means that ground estimations of BRDF can be aggregated to coarser resolutions (i.e., MISR 250 m) by calculating the area-weighted average of the kernel weights. The MISR-derived and up-scaled BRDF model kernel weights can be directly compared; and the field-derived area-weighed kernel weights can be used to predict the red band BRF in all nine MISR cameras.

Figure 8. Plots of red band BRF as a function of view zenith angle in the principal plane for site six: (a) BRF for sampling point 02, (b) BRF for sampling point 03, (c) BRF for sampling point 04, (d) BRF for sampling point 07. Negative viewing zenith angles indicate forward-scattering and positive values indicate backward-scattering. Continued…
Initial Shrub Cover Estimates from MISR/GO and RTLS-R Kernel Weights

In 2012 we used our extensive validation data to calibrate background BRDF prediction equations in MISR/GO model inversion efforts and to examine relationships that would allow empirically-predicted mapping of tall shrub cover. We were also able to compare our results with data from two recently-published raster maps of fractional cover for all shrubs and tall shrubs (height > 1 m) for the North Slope of Alaska, for the year 2000, based on interpretation of 30 m Landsat imagery using a Random Forest regression algorithm (Beck et al. 2011). These are the first “circa 2000 baseline” shrub cover maps ever produced and are a very useful reference, although there are some obvious limitations. These maps are hereafter referred to as “C2B”. Efforts to exploit the RTLS-R kernel weights in shrub cover mapping were based partly on observations of a notable spatial correspondence between the C2B maps and false color composites generated using kernel weights (see: http://csam.montclair.edu/~chopping/tundra/misr_landsat_comps.php).

To evaluate the 30 m “all shrub” and “tall shrub” maps we calculated minimum, maximum, mean and standard deviation statistics for all map cells falling inside each of our 418 CANAPI+Field validation site areas (each corresponding to a 250 × 250 m MISR pixel). Mean C2B cover values > 100 or maximum C2B cover values = 255 were excluded (the maps are 8-bit with 255 used to mask missing/bad data). This resulted in N = 240 for all shrub map and N = 258 for the tall shrub map. The linear correlations between the 2010 reference cover values within the 250 m MISR cells with the ~2000 tall and all shrub map mean values are rather weak (R² = 0.28 and 0.17, respectively), with relatively high Root Mean Square Error values (0.14 and 0.80, respectively). As expected, the “all shrub” cover values are overestimated with respect to the reference data (that are for shrubs > 50 cm). The tall shrub cover values are underestimated (Figure 9 (a)), although of course this may reflect an increase in tall shrub cover over the intervening ten year period.

Figure 7. Continued… Plots of red band BRF as a function of view zenith angle in the principal plane for site six: (e) BRF for sampling point 10, (f) BRF for sampling point 12, (g) BRF for sampling point 13. Negative viewing zenith angles indicate forward-scattering and positive values indicate backward-scattering.
The retrieved MISR/GO cover values show no correlation with the reference data, although they are of a reasonable magnitude. This is likely owing to the difficulty in accurately predicting the background BRDF. Beck et al. (2011) reported that the final all and tall shrub cover maps correspond well with their field measurements, with $R^2 = 0.72$ and 0.63, respectively, and RMSE of 0.17 and 0.23, respectively; however, the validation data set had a rather small $N$: only 24 sites. Inspection of the maps reveals some strong spatial correlations with the MISR/GO cover map, the MISR iso-vol-geo kernel weight composite, and the MISR nadir camera standard false color composite (Figures 10 and 11 (b)(c) and (f), respectively). This spatial correspondence does not appear to translate to a good match with reference data and there are both similar and dissimilar features in the Landsat- and MISR/GO-based maps (Figure 12). However, fractional shrub cover maps based on RTLS-R kernel-weight Z scores and RTLS-R kernel weight Z scores plus nadir camera BRFs are able to predict shrub cover with $R^2$ values of 0.35 and 0.50, respectively, both yielding an RMSE of 0.04 (Figure 9 (c) and (d); Figures 10 and 11 (d) and (e); note that the 2007 Anaktuvuk burn area can be clearly distinguished in the maps shown in Figures 11, and 12). This approach was found to be the most promising one for exploitation of multi-angle remote sensing in Arctic shrub mapping and in 2013 we pursued mapping using a regression tree model in order to avoid the generalization implicit in the use of a single regression equation.
Figure 10. (a) Landsat-based 2000 All Shrub map (black through white = 0 – 100%) (b) MISR/GO cover retrievals (0.0-0.1) (c) iso, vol, geo kernel weight false color composite (d) tall shrub cover predicted using RTLS-R model kernel weights (0.0-0.3) (e) tall shrub cover predicted using MISR nadir camera BRFs plus RTLS-R model kernel weights (0.0-0.3) (f) MISR nadir camera standard false color composite (RGN=NRG). In (d) and (e) surface water is masked to cyan/blue using N < 0.15.
Figure 11. (a) Landsat-based 2000 All Shrub map (black through white = 0 – 100%) (b) MISR/GO cover retrievals (0.0-0.1) (c) iso, vol, geo kernel weight false color composite (d) tall shrub cover predicted using RTLS-R model kernel weights (0.0-0.3) (e) tall shrub cover predicted using MISR nadir camera BRFs plus RTLS-R model kernel weights (0.0-0.3) (f) MISR nadir camera standard false color composite (RGN=NRG). The red dots indicated the locations of the 418 250 x 250 m reference sites. In (d) and (e) surface water is masked to cyan/blue using N < 0.15.
Figure 12. (a) Landsat-based 2000 All Shrub map (black through white = 0 – 100%) (b) MISR/GO 2010 tall shrub cover retrievals (0.0-0.1) (c) Landsat-based 2000 Tall Shrub map (0 – 100%) (d) MISR/GO 2010 tall shrub means height retrievals (0 – 1 m). Mean is over 250 m cell. The red irregular polygon indicates the area of the 2007 Anaktuvuk burn. Red ovals (boxes) indicate areas of agreement (disagreement).
**MODIS/GO Model Inversions using Late Season Data**

We examined the use of MODIS BRF data from later in the season (August 20 – September 10) initially in order to determine whether it is possible to exploit the greater shrub/grass brightness contrast in the red band at this time of year. Photographs from our field campaign of August 2009 indicate that tall shrubs remain green after tussock grasses and most prostrate shrubs have become senescent (Figure 13).

![Figure 13. Field photograph in mid-August 2009 showing the contrast between tall shrubs and lower plants](image)

We performed the same procedure for GO model inversion as with MISR BRFs (using kernel-weight-predicted non-shrub background BRDFs and adjusting the mean radius and number density parameters), for both the MODIS red and NIR band data. The red band inversions were extremely noisy, with artifacts in both the cover and height maps. The model adjustment against NIR band BRFs more faithfully reflected landscape features (Figure 14) and the magnitudes of cover and height values are consistent with the expected ranges (~0.0 – 0.15; ~0 – 2 m). These results may reflect the lower level of attenuation of the signal by the atmosphere in the NIR wavelengths relative to the red wavelengths. However, since the red band inversion results were so poor, we did not pursue a full validation using the reference database, choosing instead to spend time examining the ability of RTLS-R kernel weights to predict shrub cover using empirical methods, as described above.
Figure 14. Cover and height maps from GO model inversions using MODIS 250 m BRF data for August 20 – September 10, 2010 (DOY 232-253) (a) fractional cover map from red band (black to white = 0.0 – 0.3) (b) fractional cover map from NIR band (0.0 – 0.3) (c) mean shrub height map from red band (black to white = 0.0 – 4 m) (d) mean shrub height map from NIR band (black to white = 0.0 – 4 m). Note that many of the small bright areas in the north-central part are small lakes.
North Slope of Alaska: Tall Shrub Maps from MISR

Fractional tall shrub cover on N. Slope of Alaska was mapped for 2000-2010 using MISR nadir camera blue, green, red, and NIR surface BRFs; MISR RTLS-R red band kernel weights, white sky albedo, and NBAR45 (nadir-equivalent, BRDF-adjusted reflectance); latitude; and elevation and slope from the National Elevation Dataset as predictor variables. The MISR Level 1B2 Terrain radiance scenes used were from the period 15 June – 31 July (mid-growing season), 2000-2001 and 2010 and were converted to full resolution surface BRF using MISR Toolkit routines that exploit the atmospherically-corrected MISR 1 km LAND Product BRFs. The red band data in all available cameras were used to invert the RTLS-R BRDF model to retrieve and output kernel weights, model-fitting RMSE, white and black sky albedo, nadir-equivalent BRDF-adjusted reflectance (NBAR45); and Weights of Determination for all terms.

The predictor data were filtered for cloud and surface water (using model-fitting RMSE and nadir NIR BRF), and burned areas (using the MODIS Burned Area product for this period) and interpreted using a Boosted Regression Tree (BRT) model, an ensemble method. The BRT model was trained with our reference database that leverages field and image-based information on the shrub canopy (intermediate version). The database was divided into training and validation data sets and the trained model was used with the predictor variables listed above to predict fractional cover for all locations where valid MISR data were available. The initial model had 21 predictor variables but after simplifying it, a final model with 14 predictor variables ($R^2 = 0.52$) was used (Figure 16).

Caveats: The resulting maps suffer from gaps where there is missing data. This is mostly owing to the extremely cloudy nature of this region and also to the very conservative nature of albedo and cloud flags in the MISR LAND product that is used with custom MISR Toolkit routines to calculate surface BRFs on a 250 m grid. To mitigate this, we were obliged to take data from both 2000 and 2001 to make the earlier map. The maximum fractional cover at 250 m scale is 0.22 and the shrub growth rate in the Arctic is estimated at 0.4% per year: if in a decade the shrub fractional cover increases by 4% and considering that the standard error of the BRT model was 0.024, to be sure of detecting a real change it must be $\geq 0.025$. However, because of the quilting effect in the input data to the BRT model, the changes isolated using these maps may be spurious. For this reason we adopted a median NIR BRF compositing rule for subsequent pan-Arctic tundra mapping efforts. This rule attempts to select an acceptable (non-cloud/water/burned) observation from near the middle of the series (Potapov et al. 2011).
Figure 16. Tall shrub maps for the North Slope of Alaska (a) composited fractional cover map for 2000-2001 (b) composited fractional cover map for 2010 (c) difference between composited fractional tall shrub cover maps for 2000-2001 and 2010. These maps suffer from a quilting effect resulting from the compositing process that dramatically changes fractional cover estimates; we therefore adopted a median selection rule for compositing for our pan-Arctic tundra mapping.
The five variables that contributed to the model the most were slope, red reflectance, isotropic kernel, near-infrared reflectance, white sky albedo, and NBAR45 (Fig. 17). From these plots, we can see that as the latitude increases, the shrub cover decreases; this means shrub cover decreases southward (Fig. 17c). Shrubs also are more prominent in steeper slopes (Fig. 17d). Variations of reflectance values for the red band show that where fractional cover is higher, red reflectance is lower (Fig. 17e). White sky albedo seems to be lower at higher fractional cover values (Fig. 17h) and the isotropic kernel show that the higher the fractional cover, the darker the area (Fig. 17f).

![Graphs showing contributions of variables to the model.](image)

**Figure 17.** Predictor variables against the fitted function from the BRT model. (a) latitude for initial BRT model, (b) elevation for initial BRT model, (c–i) correspond to the seven variables with the highest contribution to the BRT model. (c) latitude, (d) slope, (e) red reflectance, (f) isotropic kernel, (g) near infrared reflectance, (h) white sky albedo, (i) NBAR45. Values inside the parenthesis represent the percentage contribution of the variable to the model. The patterns in (a) and (b) do not correspond to the observed characteristics of the area, thus they were adjusted in the model to more closely represent observed patterns.

**Towards Pan-Arctic Shrub Maps from MISR**

We pursued tall shrub mapping for the entire Arctic using MISR nadir multispectral and multi-angle red band BRFs for 2000 and 2010 (Terra satellite paths #1 through #200: 1633 HDF files for 2000 and 1697 HDF files for 2010). We had already mapped the North Slope of Alaska using the BRT model, and acquired additional MISR data from June 15 – July 31 2000 and 2010 to cover parts of Canada, Scandinavia, and Russia. The magnitude of the effort required is illustrated by Fig. 18 that shows the sequence of operations required to extract and process MISR BRFs for GO model inversion or Boosted Regression Tree prediction; and Fig. 19 that shows the coverage of the...
first run with selected MISR orbits/paths/blocks northern part of Scandinavia and eastern Russia (nadir camera false color composite; height/cover retrievals via MISR/GO model inversion; and model-fitting RMSE).

Fig. 18. Flowchart of MISR data extraction and processing operations for GO model inversion (a similar sequence is performed for empirical modeling but with compositing performed prior to prediction). Custom routines are used to extract up to 6 MISR blocks at a time, convert to surface BRF and resample to a 250 m Lambert Azimuthal Equal Area grid. The data are then reformatted to ASCII files that can be used to invert the BRDF and GO models. Filtering operations are performed to isolate data contaminated by clouds, surface water, and burned area and compositing proceeds by selecting the lowest RMSE (GO model) or the overpass with the median NIR BRF (empirical models).

Figure 19. Composited MISR/GO maps of the northern part of Scandinavia and eastern Russia (a) MISR NRG False Color Composite (b) mean canopy height map (c) fractional tree cover map (d) GO model-fitting RMSE.
Tundra-Taiga Interface Maps from MISR

The changing tundra-taiga interface is of considerable interest with rapid climate warming at northern high latitudes. The taiga-tundra ecotone extends over 13,400 km and marks the transition between the northern limits of forests and the southern margin of the tundra. Mapping taiga canopies allows monitoring of displacements in the taiga-tundra boundary, that is undergoing rapid change from both natural and anthropogenic disturbance drivers, with warming occurring at a faster rate in high latitudes than anywhere else. As a result, Arctic vegetation zones are very likely to shift with wide-ranging impacts. The tree-line is expected to move northward and to higher elevations, with forests replacing tundra; vegetation is likely to increase carbon uptake, lowering land surface albedo; insect outbreaks and forest fires are very likely to increase in frequency, severity, and duration; and agriculture will have the potential to expand northward due to a longer and warmer growing season, where soils and drainage are suitable. However, the rate, extent, and direction of changes are difficult to predict because the future trajectories of environmental and disturbance factors (precipitation, drought, flood, insect response) are not well constrained or spatially uniform; observations of the tundra-taiga interface are thus required.

A first study was performed using geometric-optical canopy reflectance model inversion for canopy mapping in the taiga-tundra transition landscapes of the central Kola Peninsula, Russia (Chopping 2012). The Kola Peninsula is almost completely to the north of the Arctic Circle but has unusually warm winter temperatures for these latitudes (66 – 69° N), as it receives warmth from the North Atlantic Drift. Mapping was accomplished via inversion of the Simple Geometric-optical Model (SGM) against MISR BRFs (MISR red band BRFs for 08/06/00, Terra path 185, MISR blocks 35-36). The contribution of the background at the various Sun-target-view angles was provided by the RTLS-R BRDF model that is used in the MODIS BRDF/Albedo processing, estimated a priori using the kernel weights of the same model with regression equations calibrated against 53 high spatial resolution image chips extracted from Google Earth (e.g., Figure 20).

Figure 20. CANAPI applied to Google Earth imagery for the area corresponding to a 250 × 250 m mapped MISR pixel in the Kola Peninsula study area (a) original Google Earth imagery (north is up) (b) converted to panchromatic image (c) rotated image showing CANAPI estimates of crown locations and sizes (solar direction is up) (d) crown map rotated to the original north-up orientation.
For GO model inversion, the mean crown radius \( (r) \) and tree number density \( (\lambda, \text{trees per square meter}) \) were adjusted by minimizing the absolute root mean square error (RMSE) cost function using the Praxis optimization algorithm. This allowed retrieval of fractional crown cover and mean canopy height \( (h+b, \text{where } h \text{ is mean crown center height and } b \text{ is mean crown vertical radius}) \). In northern high latitudes, model inversion is more challenging: there is far lower contrast between the tree canopy and the background and the solar path length is greater, resulting in a lower direct:diffuse irradiance ratio.

Model inversion results were assessed with respect to reference data obtained by analysis of very high resolution imagery using CANAPI. Crown cover was retrieved accurately \( (\text{RMSE} = 0.04, R^2 = 0.65) \), with mean canopy height somewhat less so \( (\text{RMSE} = 1.9 \text{ m}, R^2 = 0.54) \). The maps of cover and mean height within 250 m cells match features in high resolution imagery (Figure 21).

The same methods were also used with data over a much larger area and for the year 2010, with similar results: although these results have not yet been validated, the spatial correspondences between the MISR/GO-based cover and mean height maps and forest density and land cover type in Google Earth imagery are clear (Figure 22).
Figure 22. (a) Landsat True Color imagery in Google Earth over the Kovdor mine and surrounding area on the Kola Peninsula (b) MSIR NRG False Color Composite (c) MISR/GO mean height map (d) MISR/GO fractional cover map. (e) zoom over the Kovdor mine area showing relatively dense forest to the north of the lake.
Dissemination of Results at Meetings

Results and related relevant research have been presented at these meetings:

Dissemination/Meetings, continued…

  http://modis.gsfc.nasa.gov/sci_team/meetings/201001/presentations/land/chopping.ppt

Dissemination of Results via the Web

Project results will be documented at the web site at http://csam.montclair.edu/~chopping/tundra. This is a repository of data outputs (digital maps, tabular data), codes (macros, scripts, modeling and inversion programs, and test data sets), publication preprints, workshop reports, presentations, and posters. Our project and research work are also recorded on the NACP website (http://www.nacarbon.org), although this record requires updating (we have published our Alaskan tundra shrub reference database for distribution.)
Publications


Anticipated forthcoming manuscripts (lead author, submission target*):

• Mapping the Tundra-Taiga Interface with MISR (Chopping, September 2015).
• Pan-Arctic woody plant abundance from MISR, 2000-2013 (Chopping, September 2015).

* The PI was incapacitated in May 2014 owing to a traffic accident, losing almost 2 months FTE.

Acknowledgments

We gratefully acknowledge the assistance of Dr. Vern Vanderbilt (NASA Ames) for advice on field radiometry and calibration target issues; Dr. K. Fred Huemmrich (UMBC), for generously sharing his field spectrometry data with us; Dr. Sawahiko Shimada (Tokyo University of Agriculture and visiting scientist at Montclair State University 2009-10) in helping to bring the Montclair-based graduate student up to speed with our MISR processing and BRDF model inversion scripts and codes. We are grateful to Dr. Lee Vierling (Geospatial Laboratory for Environmental Dynamics, University of Idaho) for kindly providing us with the PARABOLA BRF data he acquired in 1995. We also thank Pieter Beck of Woods Hole Research Center and his colleagues and co-authors for making the circa 2000 Baseline maps available. We would like to acknowledge the assistance of Joseph Youn and Michael Stoppay (CORE – Computer Operations
for Research and Education, College of Science and Mathematics, Montclair State University). We are also grateful to Xiaohong Chopping for many useful suggestions.

References


