parameters given by

$$\alpha = [0.458894, -1.617565, 2.634451, -2.367343]^T$$

$$\beta = [0.541595, -2.122080, 3.493080, -2.873419]^T$$

$$H = \begin{bmatrix} 0.051699 & 0.039181 & 0.031292 & 0.024140 \\ 0.084922 & 0.100632 & 0.088210 & 0.061256 \\ 0.105362 & 0.146137 & 0.163192 & 0.148849 \\ 0.091628 & 0.144012 & 0.197267 & 0.222232 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 1.026539 & 0.528078 & 0.865691 & 0.200961 \\ 0.441449 & 0.502900 & 0.770215 & 0.170203 \\ 0.349147 & -0.006186 & 0.789078 & 0.207311 \\ 0.086905 & -0.036714 & 0.187733 & 0.051699 \end{bmatrix}$$

where α, β, H, and Λ are defined as in (3) and (7); its impulse response is shown in Fig. 2.

V. Conclusions

The problem of designing 2-D recursive digital filters has been considered using a canonic local state-space model which is separable in the denominator. The problem has been to minimize a weighted sum of the 1_p differences between the desired and actual impulse responses over a finite interval. A procedure to reasonably find the initial model has been developed using the Cayley–Hamilton theorem, which is very crucial to attain a good convergent accuracy in the nonlinear optimization problem. A new mapping from an asymmetric half-plane to a subset of the first quadrant has also been proposed to adjust the order of numerator and to design a wider class of 2-D filters. This can be viewed as a generalization of the mapping reported in [7], [8], and [15]. Two examples have shown that the resulting filters have almost the same or better accuracy than the others by the existing techniques in many cases.

If f_{ij} is used instead of h_{ij} in (2) as the entries of $c_1^{(0)}$, $b_2^{(0)}$, and $A_3^{(0)}$, the computational burden involved in (13) can be eased at the expense of accuracy. Unless (2) is given in a canonic form, the number of parameters becomes $n^2 + m^2 + nm + 2(n+m)+1$ in general. This implies that the use of a canonic form is effective to reduce the computational burden involved in (14). The appropriate choice of the coordinates for the resulting canonic form gives a realization with minimum roundoff noise and no overflow oscillations [16]. It is interesting to note that a class of 2-D separable-denominator filters nicely approximates circularly symmetric magnitude responses [17]. Notice that the canonic form in (2) always guarantees the minimality [10], [18] and is stable provided all the eigenvalues of A_1 and A_2 exist inside the unit disk. Any suitable nonlinear optimization method [14] is applicable to minimize (5).

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State-Space Realization of 2-D Systems via Continued Fraction Expansion

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***Abstract*—An algorithm is presented for minimal state-space realization of two-dimensional (2-D) systems. The method is based on the idea of expanding the transfer function in a 2-D continued fraction. The algorithm proposed in conceptually and computationally simple.**

I. Introduction

Consider the linear time invariant two-dimensional (2-D) system described by the transfer function

$$T(w,z) = \frac{\sum_{i=0}^{m} \sum_{j=0}^{n} g_{ij} w^i z^j}{\sum_{i=0}^{m} \sum_{j=0}^{n} h_{ij} w^i z^j}. \tag{1}$$

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The problem considered in this paper is to derive for (1) a minimal state-space model, i.e., a model whose state vector will be of dimension $m+n$. The particular state-space model sought is of the following form [1], [2]:

$$\overset{*}{x}(i,j) = Ax(i,j) + bu(i,j)$$
$$y(i,j) = c^T x(i,j) + du(i,j) \tag{2}$$

where

$$\overset{*}{x}(i,j) = \begin{bmatrix} x^h(i+1,j) \\ \hline x^v(i,j+1) \end{bmatrix}, \quad x(i,j) = \begin{bmatrix} x^h(i,j) \\ \hline x^v(i,j) \end{bmatrix}$$

where $x(i,j) \in R_{n+m}$, $u(i,j) \in R_1$, $y(i,j) \in R_1$, and A, b, c^T have appropriate dimensions and are partitioned accordingly as

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \quad b = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

and

$$c^T = [\, c_{11} \mid c_{12} \,].$$

The problem considered here has already been studied in the past. Kung *et al.* [3] have proposed a simple realization method, but the resulting state-space model is nonminimal since the dimension of the state-space vector is $2m+n$. A different state-space model was reported by Fornasini–Marchesini [4], [5]. This model is also nonminimal since the state-space vector is of dimension $2|\max(m,n)|^2$. Also, Eising's model [6] has dimensions, in general, greater than that of Kung's model. The results reported by Galkowski [7] also give nonminimal realizations.

Minimal state-space realizations of 2-D systems have recently been reported by Koonar–Sohal [8]. Their algorithm, however, appears to be complicated since it involves the determination of Hankel matrices from Markov parameters or moments of impulse response. Also in [9], minimal 2-D state-space realizations are reported for the special case where system (1) is an all-pole 2-D transfer function.

In this paper, new results are presented for a minimal 2-D state-space realization of system (1), provided that (1) can be expanded to 2-D continued fraction [10], [11]. Our approach is essentially an extension of the state-space realization approach for the case of one-dimensional (1-D) systems [12]. The proposed algorithm is conceptually and computationally simple.

II. Solution of the Problem

To solve the problem, assume without loss of generality that in (1) $m=n=2$, i.e.,

$$T(w,z) = \frac{\sum_{i=0}^{2}\sum_{j=0}^{2} g_{ij} w^i z^j}{\sum_{i=0}^{2}\sum_{j=0}^{2} h_{ij} w^i z^j}. \tag{3}$$

Assume that system (3) may be expanded in one of the 2-D continued fraction types suggested in [11] and [13]. In particular, assume that (3) is expanded in the III-A-type [11], [14] having the general form

$$T(w,z) = \cfrac{1}{c_1 + \cfrac{1}{A_1 w + \cfrac{1}{c_2 + \cfrac{1}{B_1 z + \cfrac{1}{c_3 + \cfrac{1}{A_2 w + \cfrac{1}{c_4 + \cfrac{1}{B_2 z + \cfrac{1}{c_5}}}}}}}}} \tag{4}$$

Using (4), $T(w,z)$ may be easily presented in a block diagram form as in Fig. 1, by extending the results of 1-D systems [12] to 2-D systems.

From Fig. 1, the following state-space model may be written down by inspection:

$$L\overset{*}{x}(i,j) = Mx(i,j) + eu(i,j)$$
$$y(i,j) = c^T x(i,j) + du(i,j) \tag{5}$$

where

$$\overset{*}{x}(i,j) = \begin{bmatrix} x_1^h(i+1,j) \\ x_2^h(i+1,j) \\ \hline x_1^v(i,j+1) \\ x_2^v(i,j+1) \end{bmatrix}, \quad x(i,j) = \begin{bmatrix} x_1^h(i,j) \\ x_2^h(i,j) \\ \hline x_1^v(i,j) \\ x_2^v(i,j) \end{bmatrix}$$

$$L = \left[\begin{array}{cc|cc} c_1 A_1 & c_1 A_2 & c_1 B_1 & c_1 B_2 \\ 0 & c_3 A_2 & 0 & c_3 B_2 \\ \hline 0 & c_2 A_2 & c_2 B_1 & c_2 B_2 \\ 0 & 0 & 0 & c_4 B_2 \end{array}\right]$$

$$M = \left[\begin{array}{cc|cc} -1 & 0 & 0 & -c_1/c_5 \\ 0 & -1 & 1 & -c_3/c_5 \\ \hline 1 & 0 & -1 & -c_2/c_5 \\ 0 & 1 & 0 & -\left(1+\dfrac{c_4}{c_5}\right) \end{array}\right]$$

$$e = \begin{bmatrix} 1 \\ 0 \\ \hline 0 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} -1/c_1 \\ 0 \\ \hline 0 \\ 1 \end{bmatrix}, \quad d = \frac{1}{c_1}.$$

Then, if L is invertible, (5) may be written as

$$\overset{*}{x}(i,j) = L^{-1}Mx(i,j) + L^{-1}eu(i,j)$$
$$y(i,j) = c^T x(i,j) + du(i,j). \tag{6}$$

Comparing (2) and (6), we readily have

$$A = L^{-1}M \quad \text{and} \quad b = L^{-1}e \tag{7}$$

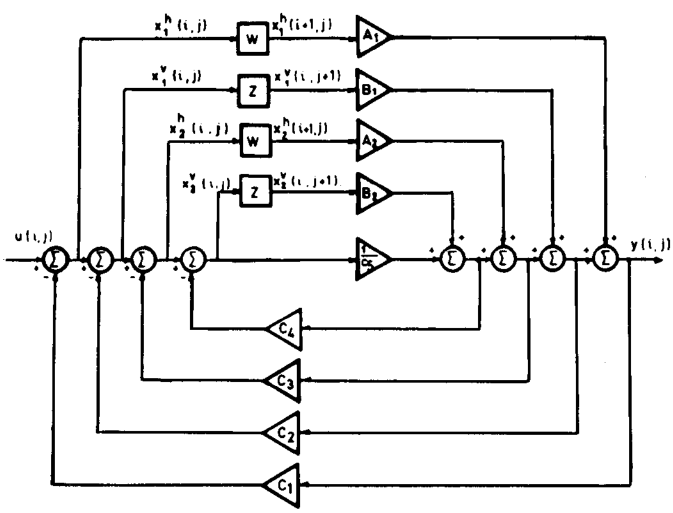

Fig. 1. Block diagram realization of (4).

where

$$A = L^{-1}M = \left[\begin{array}{cc|cc} -\dfrac{c_1+c_2}{A_1c_1c_2} & 0 & \dfrac{1}{A_1c_2} & 0 \\ 0 & -\dfrac{c_3+c_4}{A_2c_3c_4} & \dfrac{1}{A_2c_3} & \dfrac{1}{A_2c_4} \\ \hline \dfrac{1}{B_1c_2} & \dfrac{1}{B_1c_3} & \dfrac{c_2+c_3}{B_1c_2c_3} & 0 \\ 0 & \dfrac{1}{B_2c_4} & 0 & -\dfrac{c_4+c_5}{B_2c_4c_5} \end{array}\right]$$

$$b^T = L^{-1}N = \left[\begin{array}{cc|cc} \dfrac{1}{A_1c_1} & 0 & 0 & 0 \end{array}\right].$$

Now, consider the case where $T(w,z)$ is not of the full order (2,2). For this case, the determination of the state-space model may be simplified by using (6) and (7). To this end, define the $m+n=4$ diagonal square matrix Q as

$$Q = \mathrm{diag}[q_1^w, q_2^w, q_1^z, q_2^z] \tag{8}$$

where, using the notation in (4), the constants q_i^w and q_i^z are defined as

$$q_i^w = \begin{cases} 1, & \text{if } A_i \neq 0 \\ 0, & \text{if } A_i = 0 \end{cases} \quad q_i^z = \begin{cases} 1, & \text{if } B_i \neq 0 \\ 0, & \text{if } B_i = 0 \end{cases}, \quad i = 1,2.$$

Then, it can be readily derived that a state-space model, when (3) is not of full (2,2) order, is given by

$$\overset{*}{x}_r(i,j) = A_r x_r(i,j) + b_r u(i,j)$$

$$y_r(i,j) = c_r^T x_r(i,j) + d_r u(i,j) \tag{9}$$

where

$$\overset{*}{x}_r(i,j) = Q\overset{*}{x}(i,j) \quad A_r = QAQ$$

$$y_r(i,j) = y(i,j) \quad b_r = Qb$$

$$x_r(i,j) = Qx(i,j) \quad c_r^T = c^TQ \text{ and } d_r = d$$

where A and b, are specified in (7) and c^T and d in (5). Next, observe that, in (9), zero rows and zero columns will appear. For example, if $q_2^w = 0$, then the second row and second column in (9) are zero. By deleting all zero rows and zero columns in (9) we reach the final state-space model sought. This is demonstrated in the two examples that follow.

The results of this section may easily be extended to cover a system of general order (m,n) as specified in (1). Furthermore, other types of continued fraction may be used to derive state-space models.

Example 1

Consider the 2-D system described by the following transfer function given in [11]:

$$T(w,z) = \frac{15+81z+33w+144wz+12w^2+48w^2z}{2+27z+27w+120wz+12w^2+48w^2z}. \tag{10}$$

The corresponding 2-D continued fraction expansion of type III-A has the form

$$T(w,z) = \cfrac{1}{1+\cfrac{1}{-2w+\cfrac{1}{-\frac{2}{3}+\cfrac{1}{-27z+\cfrac{1}{-\frac{4}{21}+\cfrac{1}{-49w+\cfrac{1}{-105}}}}}}}. \tag{11}$$

For the present case, Q takes on the form

$$Q = \mathrm{diag}[1,1,1,0].$$

Hence

$$A_r = QAQ = \left[\begin{array}{cc|cc} -\dfrac{c_1+c_2}{A_1c_1c_2} & 0 & \dfrac{1}{A_1c_2} & 0 \\ 0 & -\dfrac{c_3+c_4}{A_2c_3c_4} & \dfrac{1}{A_2c_3} & 0 \\ \hline \dfrac{1}{B_1c_2} & \dfrac{1}{B_1c_3} & -\dfrac{c_2+c_3}{B_1c_2c_3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} -\dfrac{1}{4} & 0 & \dfrac{3}{4} & 0 \\ 0 & -\dfrac{9}{4} & \dfrac{21}{196} & 0 \\ \hline \dfrac{3}{54} & \dfrac{21}{108} & -\dfrac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$b_r = Qb = \begin{bmatrix} \dfrac{1}{A_1c_1} \\ 0 \\ \hline 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2} \\ 0 \\ \hline 0 \\ 0 \end{bmatrix}$$

$$c_r^T = c^TQ = \left[\begin{array}{cc|cc} -\dfrac{1}{c_1} & 0 & 0 & 0 \end{array}\right] = [-1 \quad 0 \mid 0 \quad 0]$$

$$d_r = d = \frac{1}{c_1} = 1$$

$$\overset{*}{x}_r(i,j)=Q\overset{*}{x}(i,j)=\left[\begin{array}{c} x_1^h(i+1,j)\\ x_2^h(i+1,j)\\ \hline x_1^v(i,j+1)\\ 0\end{array}\right]$$

$$x_r(i,j)=Qx(i,j)=\left[\begin{array}{c} x_1^h(i,j)\\ x_2^h(i,j)\\ \hline x_1^v(i,j)\\ 0\end{array}\right].$$

By deleting the zero row and zero column, we arrive at the final state-space model

$$\left[\begin{array}{c} x_{1r}^h(i+1,j)\\ x_{2r}^h(i+1,j)\\ \hline x_{1r}^v(i,j+1)\end{array}\right]=\left[\begin{array}{cc|c} -\frac{1}{4} & 0 & \frac{3}{4}\\ 0 & -\frac{9}{4} & \frac{21}{196}\\ \hline \frac{3}{54} & \frac{21}{108} & -\frac{1}{4}\end{array}\right]\left[\begin{array}{c} x_{1r}^h(i,j)\\ x_{2r}^h(i,j)\\ \hline x_{1r}^v(i,j)\end{array}\right]+\left[\begin{array}{c} -\frac{1}{2}\\ 0\\ \hline 0\end{array}\right]u_r(i,j)$$

$$y_r(i,j)=\left[\begin{array}{cc|c} -1 & 0 & 0\end{array}\right]\left[\begin{array}{c} x_{1r}^h(i,j)\\ x_{2r}^h(i,j)\\ \hline x_{1r}^v(i,j)\end{array}\right]+u_r(i,j). \tag{12}$$

Checking, we have that the transfer function of (12) is exactly equal to (10).

Example 2

Consider the 2-D system described by the following transfer function given in [15]:

$$T(w,z)=\frac{5+8z+8w+12wz}{1+2z+8w+12wz}. \tag{13}$$

The corresponding 2-D continued fraction expansion of type III-A has the form

$$T(w,z)=\cfrac{1}{1+\cfrac{1}{-2w+\cfrac{1}{-\frac{3}{4}+\cfrac{1}{-32z+\cfrac{1}{-\frac{1}{20}}}}}}. \tag{14}$$

For the present case Q takes the form

$$Q=\operatorname{diag}[1,0,1,0].$$

Hence

$$A_r=QAQ=\left[\begin{array}{cc|cc} -\frac{c_1+c_2}{A_1c_1c_2} & 0 & \frac{1}{A_1c_2} & 0\\ 0 & 0 & 0 & 0\\ \hline \frac{1}{B_1c_2} & 0 & -\frac{c_2+c_3}{B_1c_2c_3} & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

$$=\left[\begin{array}{cccc} -\frac{1}{6} & 0 & \frac{2}{3} & 0\\ 0 & 0 & 0 & 0\\ \hline \frac{1}{24} & 0 & -\frac{2}{3} & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

$$b_r=Qb=\left[\begin{array}{c} \frac{1}{A_1c_1}\\ 0\\ \hline 0\\ 0\end{array}\right]=\left[\begin{array}{c} -\frac{1}{2}\\ 0\\ \hline 0\\ 0\end{array}\right]$$

$$c_r^T=c^TQ=\left[\begin{array}{cc|cc} -\frac{1}{c_1} & 0 & 0 & 0\end{array}\right]=\left[\begin{array}{cc|cc} -1 & 0 & 0 & 0\end{array}\right]$$

$$d_r=d=\frac{1}{c_1}=1$$

$$\overset{*}{x}_r=Q\overset{*}{x}(i,j)=\left[\begin{array}{c} x_1^h(i+1,j)\\ 0\\ \hline x_1^v(i,j+1)\\ 0\end{array}\right]$$

$$x_r=Qx(i,j)=\left[\begin{array}{c} x_1^h(i,j)\\ 0\\ \hline x_1^v(i,j)\\ 0\end{array}\right].$$

By deleting the zero rows and zero columns, we arrive at the state-space model sought, and finally (9) may be written as

$$\left[\begin{array}{c} x_{1r}^h(i+1,j)\\ \hline x_{1r}^v(i,j+1)\end{array}\right]=\left[\begin{array}{c|c} -\frac{1}{6} & \frac{2}{3}\\ \hline \frac{1}{24} & -\frac{2}{3}\end{array}\right]\left[\begin{array}{c} x_{1r}^h(i,j)\\ \hline x_{1r}^v(i,j)\end{array}\right]=\left[\begin{array}{c} -\frac{1}{2}\\ \hline 0\end{array}\right]u_r(i,j)$$

$$y_r(i,j)=\left[\begin{array}{c|c} -1 & 0\end{array}\right]\left[\begin{array}{c} x_{1r}^h(i,j)\\ \hline x_{1r}^v(i,j)\end{array}\right]+u_r(i,j). \tag{15}$$

Checking, we have found that the transfer function of (15) is exactly equal to (13).

III. Conclusions

A well-known technique for state-space realization of 1-D systems is extended for the case of 2-D systems. This technique is based on the idea of expanding the transfer function to continued fraction expansion, and subsequently writing down by inspection the state-space equations. This approach is simple and yields a minimal realization. Of course, our approach cannot be applied in cases where the transfer function is not expandable to continued fraction.

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